Name:____________________________ Class:__________________ Date:__________________

Sample Problem Set I Solutions

Two-Dimensional Motion and Vectors

ADDITIONAL PRACTICE C

Givens	*Solutions*
1. $\Delta x_1 = 250.0$ m $d_2 = 125.0$ m $\theta_2 = 120.0°$	$\Delta x_2 = d_2(\cos\theta_2) = (125.0 \text{ m})(\cos 120.0°) = -62.50 \text{ m}$ $\Delta y_2 = d_2(\sin\theta_2) = (125.0 \text{ m})(\sin 120.0°) = 108.3 \text{ m}$ $\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 250.0 \text{ m} - 62.50 \text{ m} = 187.5 \text{ m}$ $\Delta y_{tot} = \Delta y_1 + \Delta y_2 = 0 \text{ m} + 108.3 \text{ m} = 108.3 \text{ m}$ $d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(187.5 \text{ m})^2 + (108.3 \text{ m})^2}$ $d = \sqrt{3.516 \times 10^4 \text{ m}^2 + 1.173 \times 10^4 \text{ m}^2} = \sqrt{4.689 \times 10^4 \text{ m}^2}$ $d = \boxed{216.5 \text{ m}}$ $\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{108.3 \text{ m}}{187.5 \text{ m}}\right) = \boxed{30.01° \text{ north of east}}$
2. $v = 3.53 \times 10^3$ km/h $\Delta t_1 = 20.0$ s $\Delta t_2 = 10.0$ s $\theta_1 = 15.0°$ $\theta_2 = 35.0°$	$\Delta x_1 = v\Delta t_1(\cos\theta_1)$ $\Delta x_1 = (3.53 \times 10^3 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)(10^3 \text{ m/km})(20.0 \text{ s})(\cos 15.0°) = 1.89 \times 10^4 \text{ m}$ $\Delta y_1 = v\Delta t_1(\sin\theta_1)$ $\Delta y_1 = (3.53 \times 10^3 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)(10^3 \text{ m/km})(20.0 \text{ s})(\sin 15.0°) = 5.08 \times 10^3 \text{ m}$ $\Delta x_2 = v\Delta t_2(\cos\theta_2)$ $\Delta x_2 = (3.53 \times 10^3 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)(10^3 \text{ m/km})(10.0 \text{ s})(\cos 35.0°) = 8.03 \times 10^3 \text{ m}$ $\Delta y_2 = v\Delta t_2(\sin\theta_2)$ $\Delta y_2 = (3.53 \times 10^3 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)(10^3 \text{ m/km})(10.0 \text{ s})(\sin 35.0°) = 5.62 \times 10^3 \text{ m}$ $\Delta y_{tot} = \Delta y_1 + \Delta y_2 = 5.08 \times 10^3 \text{ m} + 5.62 \times 10^3 \text{ m} = \boxed{1.07 \times 10^4 \text{ m}}$ $\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 1.89 \times 10^4 \text{ m} + 8.03 \times 10^3 \text{ m} = \boxed{2.69 \times 10^4 \text{ m}}$ $d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(2.69 \times 10^4 \text{ m})^2 + (1.07 \times 10^4 \text{ m})^2}$ $d = \sqrt{7.24 \times 10^8 \text{ m}^2 + 1.11 \times 10^8 \text{ m}^2} = \sqrt{8.35 \times 10^8 \text{ m}}$ $d = \boxed{2.89 \times 10^4 \text{ m}}$ $\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{1.07 \times 10^4 \text{ m}}{2.69 \times 10^4 \text{ m}}\right)$ $\theta = \boxed{21.7° \text{ above the horizontal}}$

Givens	Solutions
3. $\Delta x_1 + \Delta x_2 = 2.00 \times 10^2$ m $\Delta y_1 + \Delta y_2 = 0$ $\theta_1 = 30.0°$ $\theta_2 = -45.0°$ $v = 11.6$ km/h	$\Delta y_1 = d_1(\sin\theta_1) = -\Delta y_2 = -d_2(\sin\theta_2)$ $d_1 = -d_2\left(\frac{\sin\theta_2}{\sin\theta_1}\right) = -d_2\left[\frac{\sin(-45.0°)}{\sin 30.0°}\right] = 1.41d_2$ $\Delta x_1 = d_1(\cos\theta_1) = (1.41d_2)(\cos 30.0°) = 1.22d_2$ $\Delta x_2 = d_2(\cos\theta_2) = d_2[\cos(-45.0°)] = 0.707d_2$ $\Delta x_1 + \Delta x_2 = d_2(1.22 + 0.707) = 1.93d_2 = 2.00 \times 10^2$ m $d_2 = \boxed{104 \text{ m}}$ $d_1 = (1.41)d_2 = (1.41)(104 \text{ m}) = \boxed{147 \text{ m}}$ $v = 11.6 \text{ km/h} = (11.6 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)(10^3 \text{ m/km}) = 3.22 \text{ m/s}$ $\Delta t_1 = \frac{d_1}{v} = \left(\frac{147 \text{ m}}{3.22 \text{ m/s}}\right) = 45.7 \text{ s}$ $\Delta t_2 = \frac{d_2}{v} = \left(\frac{104 \text{ m}}{3.22 \text{ m/s}}\right) = 32.3 \text{ s}$ $\Delta t_{tot} = \Delta t_1 + \Delta t_2 = 45.7 \text{ s} + 32.3 \text{ s} = \boxed{78.0 \text{ s}}$
4. $v = 925$ km/h $\Delta t_1 = 1.50$ h $\Delta t_2 = 2.00$ h $\theta_2 = 135°$	$d_1 = v\Delta t_1 = (925 \text{ km/h})(10^3 \text{ m/km})(1.50 \text{ h}) = 1.39 \times 10^6$ m $d_2 = v\Delta t_2 = (925 \text{ km/h})(10^3 \text{ m/km})(2.00 \text{ h}) = 1.85 \times 10^6$ m $\Delta x_1 = d_1 = 1.39 \times 10^6$ m $\Delta y_1 = 0$ m $\Delta x_2 = d_2(\cos\theta_2) = (1.85 \times 10^6 \text{ m})(\cos 135°) = -1.31 \times 10^6$ m $\Delta y_2 = d_2(\sin\theta_2) = (1.85 \times 10^6 \text{ m})(\sin 135°) = 1.31 \times 10^6$ m $\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 1.39 \times 10^6 \text{ m} + (-1.31 \times 10^6 \text{ m}) = 0.08 \times 10^6$ m $\Delta y_{tot} = \Delta y_1 + \Delta y_2 = 0 \text{ m} + 1.31 \times 10^6 \text{ m} = 1.31 \times 10^6$ m $d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(0.08 \times 10^6 \text{ m})^2 + (1.31 \times 10^6 \text{ m})^2}$ $d = \sqrt{6 \times 10^9 \text{ m}^2 + 1.72 \times 10^{12} \text{ m}^2} = \sqrt{1.73 \times 10^{12} \text{ m}^2}$ $d = \boxed{1.32 \times 10^6 \text{ m} = 1.32 \times 10^3 \text{ km}}$ $\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{1.31 \times 10^6 \text{ m}}{0.08 \times 10^6 \text{ m}}\right) = 86.5° = 90.0° - 3.5°$ $\theta = \boxed{3.5° \text{ east of north}}$

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Givens	Solutions
5. $v = 57.2$ km/h	$d_1 = v\Delta t_1 = (57.2 \text{ km/h})(2.50 \text{ h}) = 143 \text{ km}$
$\Delta t_1 = 2.50$ h	$d_2 = v\Delta t_2 = (57.2 \text{ km/h})(1.50 \text{ h}) = 85.8 \text{ km}$
$\Delta t_2 = 1.50$ h	$\Delta_{tot} = d_1 + d_2(\cos\theta_2) = 143 \text{ km} + (85.8 \text{ km})(\cos 30.0°) = 143 \text{ km} + 74.3 \text{ km} = 217 \text{ km}$
$\theta_2 = 30.0°$	$\Delta y_{tot} = d_2(\sin\theta_2) = (85.8 \text{ km})(\sin 30.0°) = 42.9 \text{ km}$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(217 \text{ km})^2 + (42.9 \text{ km})}$$

$$d = \sqrt{4.71 \times 10^4 \text{ km}^2 + 1.84 \times 10^3 \text{ km}^2} = \sqrt{4.89 \times 10^4 \text{ km}^2}$$

$$d = \boxed{221 \text{ km}}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{42.9 \text{ km}}{217 \text{ km}}\right) = \boxed{11.2° \text{ north of east}}$$

Two-Dimensional Motion and Vectors

Problem C

ADDING VECTORS ALGEBRAICALLY

PROBLEM

The record for the longest nonstop closed-circuit flight by a model airplane was set in Italy in 1986. The plane flew a total distance of 1239 km. Assume that at some point the plane traveled 1.25×10^3 m to the east, then 1.25×10^3 m to the north, and finally 1.00×10^3 m to the southeast. Calculate the total displacement for this portion of the flight.

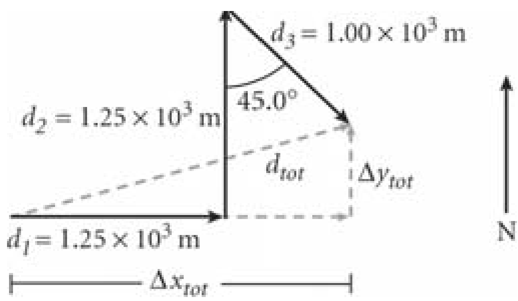
SOLUTION

1. **DEFINE**

Given: $d_1 = 1.25 \times 10^3$ m $\quad d_2 = 1.25 \times 10^3$ m $\quad d_3 = 1.00 \times 10^3$ m

Unknown: $\Delta x_{tot} = ?$ $\Delta y_{tot} = ?$ $\quad d = ?$ $\quad \theta = ?$

Diagram:



2. **PLAN** **Choose the equation(s) or situation:** Orient the displacements with respect to the x-axis of the coordinate system.

$\theta_1 = 0.00°$ $\quad \theta_2 = 90.0°$ $\quad \theta_3 = -45.0°$

Use this information to calculate the components of the total displacement along the x-axis and the y-axis.

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 + \Delta x_3$$
$$= d_1(\cos\theta_1) + d_2(\cos\theta_2) + d_3(\cos\theta_3)$$
$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 + \Delta y_3$$
$$= d_1(\sin\theta_1) + d_2(\sin\theta_2) + d_3(\sin\theta_3)$$

Use the components of the total displacement, the Pythagorean theorem, and the tangent function to calculate the total displacement.

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} \qquad \theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right)$$

3. **CALCULATE** **Substitute the values into the equation(s) and solve:**

$$\Delta x_{tot} = (1.25 \times 10^3 \text{ m})(\cos 0°) + (1.25 \times 10^3 \text{ m})(\cos 90.0°)$$
$$+ (1.00 \times 10^3 \text{ m})[\cos(-45.0°)]$$
$$= 1.25 \times 10^3 \text{ m} + 7.07 \times 10^2 \text{ m}$$
$$= 1.96 \times 10^3 \text{ m}$$
$$\Delta y_{tot} = (1.25 \times 10^3 \text{ m})(\sin 0°) + (1.25 \times 10^3 \text{ m})(\sin 90.0°)$$
$$+ (1.00 \times 10^3 \text{ m})[\sin(-45.0°)]$$
$$= 1.25 \times 10^3 \text{ m} + 7.07 \times 10^2 \text{ m}$$
$$= 0.543 \times 10^3 \text{ m}$$

$$d = \sqrt{(1.96 \times 10^3\ \text{m})^2 + (0.543 \times 10^3\ \text{m})^2}$$

$$d = \sqrt{3.84 \times 10^6\ \text{m}^2 + 2.95 \times 10^5\ \text{m}^2} = \sqrt{4.14 \times 10^6\ \text{m}^2}$$

$$d = 2.03 \times 10^3\ \text{m}$$

$$\theta = tan^{-1}\left(\frac{0.543 \times 10^3\ \text{m}}{1.96 \times 10^3\ \text{m}}\right)$$

$$\theta = 15.5^\circ \text{ north of east}$$

4. **EVALUATE** The magnitude of the total displacement is slightly larger than that of the total displacement in the eastern direction alone.

ADDITIONAL PRACTICE

1. For six weeks in 1992, Akira Matsushima, from Japan, rode a unicycle more than 3000 mi across the United States. Suppose Matsushima is riding through a city. If he travels 250.0 m east on one street, then turns counterclockwise through a 120.0° angle and proceeds 125.0 m northwest along a diagonal street, what is his resultant displacement?
2. In 1976, the Lockheed SR-71A *Blackbird* set the record speed for any airplane: 3.53×10^3 km/h. Suppose you observe this plane ascending at this speed. For 20.0 s, it flies at an angle of 15.0° above the horizontal, then for another 10.0 s its angle of ascent is increased to 35.0°. Calculate the plane's total gain in altitude, its total horizontal displacement, and its resultant displacement.
3. Magnor Mydland of Norway constructed a motorcycle with a wheelbase of about 12 cm. The tiny vehicle could be ridden at a maximum speed of 11.6 km/h. Suppose this tiny motorcycle travels in the directions d_1 and d_2, where d_1 is 30° with the horizontal (upward and right) and d_2 is 45° with the vertical (down and to the right). Calculate d_1 and d_2, and determine how long it takes the motorcycle to reach a net displacement of 2.0×10^2 to the right.
4. The fastest propeller-driven aircraft is the Russian TU-95/142, which can reach a maximum speed of 925 km/h. For this speed, calculate the plane's resultant displacement if it travels east for 1.50 h, then turns 135° north-west and travels for 2.00 h.
5. In 1952, the ocean liner *United States* crossed the Atlantic Ocean in less than four days, setting the world record for commercial ocean-going vessels. The average speed for the trip was 57.2 km/h. Suppose the ship moves in a straight line eastward at this speed for 2.50 h. Then, due to a strong local current, the ship's course begins to deviate northward by 30.0°, and the ship follows the new course at the same speed for another 1.50 h. Find the resultant displacement for the 4.00 h period.