Preview

- Objectives
- Scalars and Vectors
- Graphical Addition of Vectors
- Triangle Method of Addition
- Properties of Vectors

Objectives

- **Distinguish** between a scalar and a vector.
- **Add** and **subtract** vectors by using the graphical method.
- **Multiply** and **divide** vectors by scalars.

Scalars and Vectors

- A **scalar** is a physical quantity that has magnitude but no direction.
  - **Examples:** speed, volume, the number of pages in your textbook
- A **vector** is a physical quantity that has both magnitude and direction.
  - **Examples:** displacement, velocity, acceleration
- In this book, scalar quantities are in *italics*. Vectors are represented by **boldface** symbols.

End of Slide

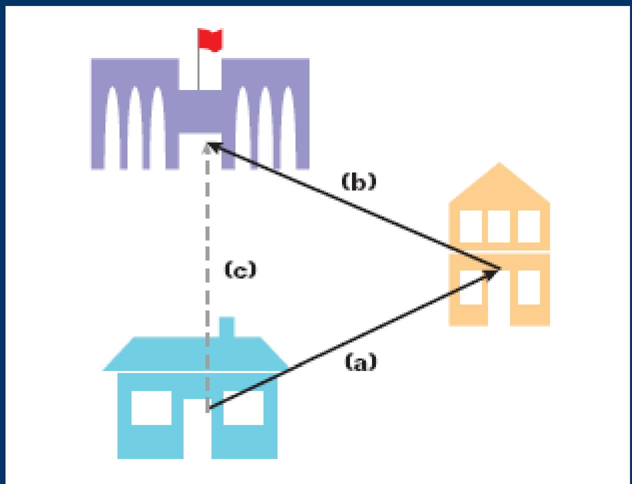
Scalars and Vectors

Click below to watch the Visual Concept.

Visual Concept

Graphical Addition of Vectors

- A **resultant vector** represents the sum of two or more vectors.
- Vectors can be added **graphically.**



*A student walks from his house to his friend's house **(a)**, then from his friend's house to the school **(b)**. The student's resultant displacement **(c)** can be found by using a ruler and a protractor.*

Triangle Method of Addition

- Vectors can be moved **parallel** to themselves in a diagram.
- Thus, you can draw one vector with its **tail** starting at the **tip** of the other as long as the size and direction of each vector do not change.
- The **resultant vector** can then be drawn from the tail of the first vector to the tip of the last vector.

Triangle Method of Addition

Click below to watch the Visual Concept.

Visual Concept

Properties of Vectors

- Vectors can be added in **any order.**
- To **subtract** a vector, add its opposite.
- **Multiplying** or **dividing** vectors by scalars results in vectors.

Properties of Vectors

Click below to watch the Visual Concept.

Visual Concept

Subtraction of Vectors

Click below to watch the Visual Concept.

Visual Concept

Multiplication of a Vector by a Scalar

Click below to watch the Visual Concept.

Visual Concept

Preview

- Objectives
- Coordinate Systems in Two Dimensions
- Determining Resultant Magnitude and Direction
- Sample Problem
- Resolving Vectors into Components
- Adding Vectors That Are Not Perpendicular

Objectives

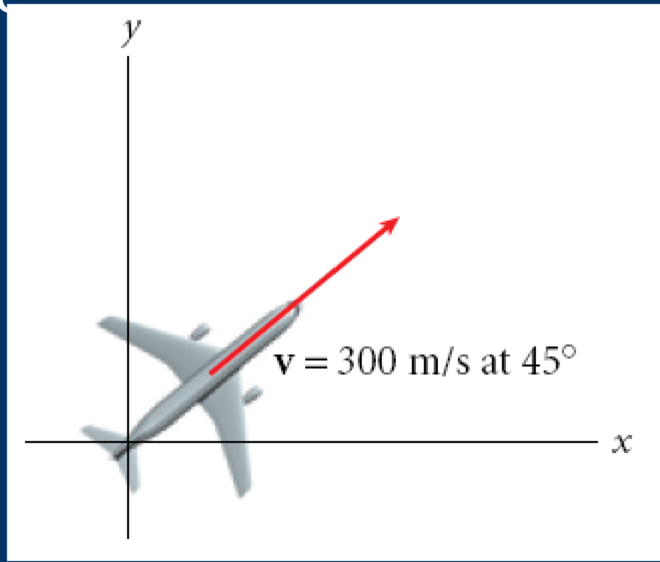
- **Identify** appropriate coordinate systems for solving problems with vectors.
- **Apply** the Pythagorean theorem and tangent function to calculate the magnitude and direction of a resultant vector.
- **Resolve** vectors into components using the sine and cosine functions.
- **Add** vectors that are not perpendicular.

Coordinate Systems in Two Dimensions

- One method for diagraming the motion of an object employs **vectors** and the use of the ***x*- and *y*-axes**.

- Axes are often designated using **fixed directions.**

- In the figure shown here, the **positive *y*-axis** points **north** and the **positive *x*-axis** points **east**.



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Determining Resultant Magnitude and Direction

- In Section 1, the magnitude and direction of a resultant were found **graphically.**
- With this approach, the accuracy of the answer depends on how carefully the diagram is drawn and measured.
- A simpler method uses the **Pythagorean theorem** and the **tangent function.**

End of Slide

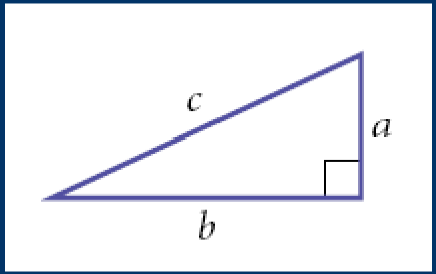
Determining Resultant Magnitude and Direction, *continued*

The Pythagorean Theorem

- Use the **Pythagorean theorem** to find the magnitude of the resultant vector.
- The Pythagorean theorem states that for any **right triangle,** the **square of the hypotenuse**—the side opposite the right angle—**equals the sum of the squares of the other two sides**, or legs.

$$c^2 = a^2 + b^2$$

$$(\text{hypotenuse})^2 = (\text{leg } 1)^2 + (\text{leg } 2)^2$$

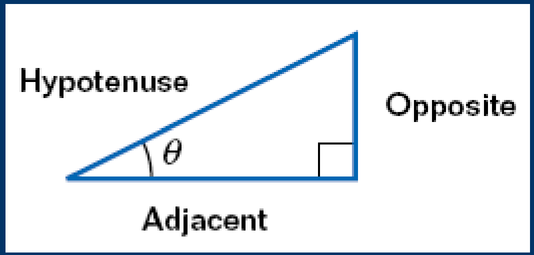


Determining Resultant Magnitude and Direction, *continued*

The Tangent Function

- Use the **tangent function** to find the direction of the resultant vector.
- For any right triangle, the **tangent** of an angle is defined as the **ratio of the opposite and adjacent legs** with respect to a **specified acute angle** of a **right triangle.**

$$\text{tangent of angle } \theta = \frac{\text{opposite leg}}{\text{adjacent leg}}$$



Sample Problem

Finding Resultant Magnitude and Direction

An archaeologist climbs the Great Pyramid in Giza, Egypt. The pyramid's height is 136 m and its width is 2.30×10^2 *m. What is the magnitude and the direction of the displacement of the archaeologist after she has climbed from the bottom of the pyramid to the top?*

End of Slide

Sample Problem, *continued*

1. Define

Given:

Δy = 136 m

Δx = 1/2(width) = 115 m

Unknown:

d = ? θ = ?

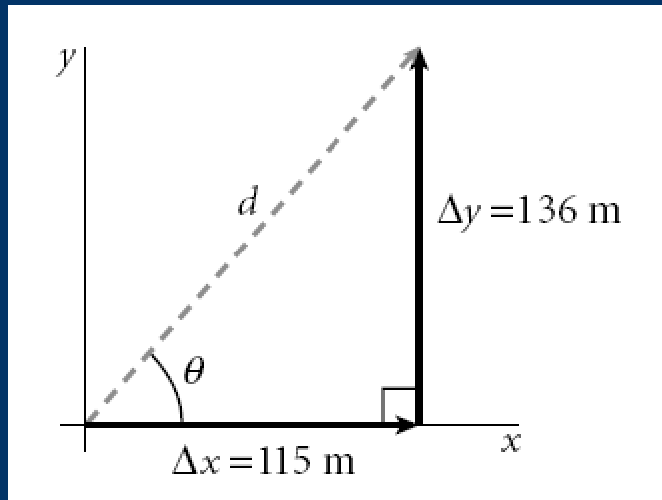


Diagram:

Choose the archaeologist's starting position as the origin of the coordinate system, as shown above.

Sample Problem, *continued*

2. Plan

Choose an equation or situation: The Pythagorean theorem can be used to find the magnitude of the archaeologist's displacement. The direction of the displacement can be found by using the inverse tangent function.

$$d^2 = \Delta x^2 + \Delta y^2 \qquad \tan\theta = \frac{\Delta y}{\Delta x}$$

Rearrange the equations to isolate the unknowns:

$$d = \sqrt{\Delta x^2 + \Delta y^2} \qquad \theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)$$

End of Slide

Sample Problem, *continued*

3. Calculate

$$d = \sqrt{\Delta x^2 + \Delta y^2} \qquad \theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)$$

$$d = \sqrt{(115\text{ m})^2 + (136\text{ m})^2} \qquad \theta = \tan^{-1}\left(\frac{136\text{ m}}{115}\right)$$

$$\boxed{d = 178\text{ m}} \qquad \boxed{\theta = 49.8^\circ}$$

4. Evaluate

Because *d* is the hypotenuse, the archaeologist's displacement should be less than the sum of the height and half of the width. The angle is expected to be more than 45° because the height is greater than half of the width.

End Of Slide

Resolving Vectors into Components

- You can often describe an object's motion more conveniently by breaking a single vector into two **components,** or **resolving the vector.**

- The **components of a vector** are the projections of the vector along the axes of a coordinate system.

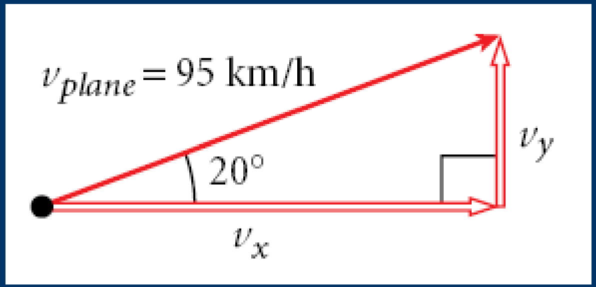
- Resolving a vector allows you to **analyze the motion in each direction.**

End of Slide

Resolving Vectors into Components, *continued*

Consider an airplane flying at 95 km/h.

- The **hypotenuse (v_{plane})** is the **resultant vector** that describes the airplane's **total velocity.**
- The **adjacent leg** represents the ***x* component** (v_x), which describes the airplane's **horizontal speed.**
- The **opposite leg** represents the ***y* component** (v_y), which describes the airplane's **vertical speed.**



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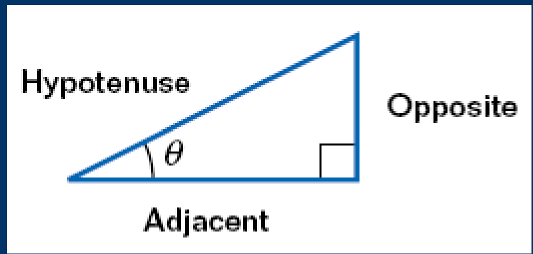
Preview
Main

Resolving Vectors into Components, *continued*

- The **sine** and **cosine functions** can be used to find the components of a vector.
- The sine and cosine functions are defined in terms of the lengths of the sides of **right triangles.**

$$\text{sine of angle } \theta = \frac{\text{opposite leg}}{\text{hypotenuse}}$$



$$\text{cosine of angle } \theta = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

End of Slide

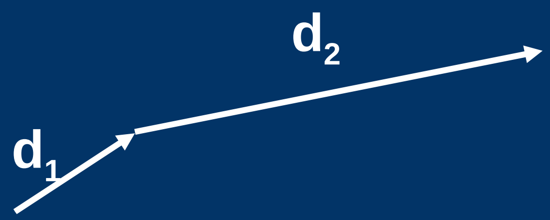
Resolving Vectors

Click below to watch the Visual Concept.

Visual Concept

Adding Vectors That Are Not Perpendicular

- Suppose that a plane travels first **5 km** at an angle of **35°,** then climbs at **10°** for **22 km,** as shown below. How can you find the **total displacement?**
- Because the original displacement vectors do not form a right triangle, you can not directly apply the tangent function or the Pythagorean theorem.



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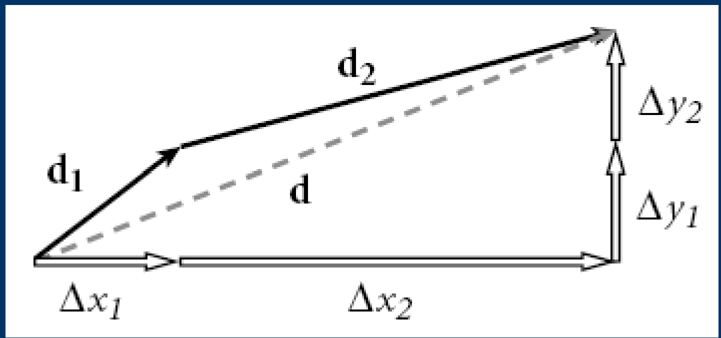
Adding Vectors That Are Not Perpendicular, *continued*

- You can find the magnitude and the direction of the resultant by resolving each of the plane's displacement vectors into its *x* and *y* components.
- Then the components along each axis can be added together.



As shown in the figure, these sums will be the two perpendicular components of the resultant, d. The resultant's magnitude can then be found by using the Pythagorean theorem, and its direction can be found by using the inverse tangent function.

End of Slide

Adding Vectors That Are Not Perpendicular

Click below to watch the Visual Concept.

Visual Concept

Sample Problem

Adding Vectors Algebraically

A hiker walks 27.0 km from her base camp at 35° south of east. The next day, she walks 41.0 km in a direction 65° north of east and discovers a forest ranger's tower. Find the magnitude and direction of her resultant displacement

End of Slide

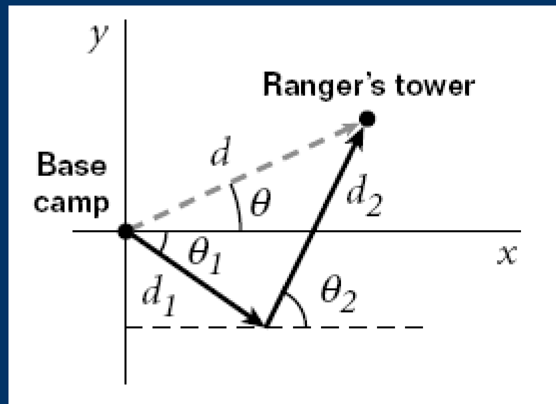
Sample Problem, *continued*

1 . Select a coordinate system. Then sketch and label each vector.

Given:

d_1 = 27.0 km $\theta_1 = -35°$

d_2 = 41.0 km $\theta_2 = 65°$



Tip: *θ_1 is negative, because clockwise movement from the positive x-axis is negative by convention.*

Unknown:

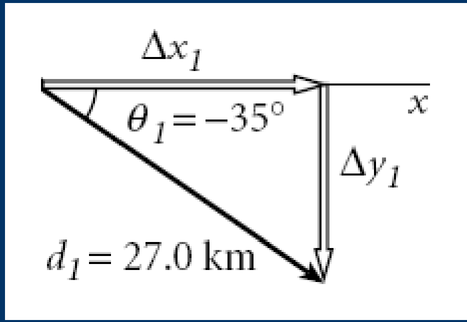
d = ? θ = ?

End Of Slide

Sample Problem, *continued*

2 . Find the *x* and *y* components of all vectors.

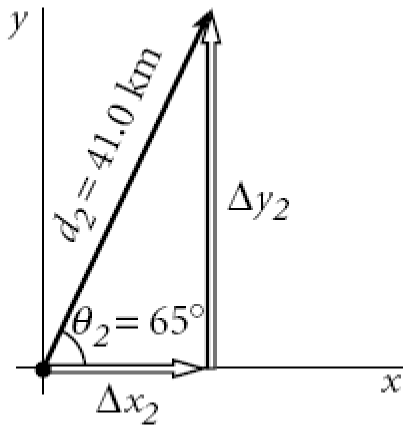
Make a separate sketch of the displacements for each day. Use the cosine and sine functions to find the components.



For day 1 :

$$\Delta x_1 = d_1 \cos \theta_1 = (27.0\ \text{km})(\cos -35°) = 22\ \text{km}$$

$$\Delta y_1 = d_1 \sin \theta_1 = (27.0\ \text{km})(\sin -35°) = -15\ \text{km}$$



For day 2 :

$$\Delta x_2 = d_2 \cos \theta_2 = (41.0\ \text{km})(\cos 65°) = 17\ \text{km}$$

$$\Delta y_2 = d_2 \sin \theta_2 = (41.0\ \text{km})(\sin 65°) = 37\ \text{km}$$

Sample Problem, *continued*

3 . Find the *x* and *y* components of the total displacement.

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 22\text{ km} + 17\text{ km} = 39\text{ km}$$

$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 = -15\text{ km} + 37\text{ km} = 22\text{ km}$$

4 . Use the Pythagorean theorem to find the magnitude of the resultant vector.

$$d^2 = (\Delta x_{tot})^2 + (\Delta y_{tot})^2$$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(39\text{ km})^2 + (22\text{ km})^2}$$

$$\boxed{d = 45\text{ km}}$$

End of Slide

Sample Problem, *continued*

5 . Use a suitable trigonometric function to find the angle.

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{22\text{ km}}{39\text{ km}}\right)$$

$$\boxed{\theta = 29^\circ \text{ north of east}}$$

End of Slide

Preview

- Objectives
- Projectiles
- Kinematic Equations for Projectiles
- Sample Problem

Objectives

- **Recognize** examples of projectile motion.
- **Describe** the path of a projectile as a parabola.
- **Resolve** vectors into their components and **apply** the kinematic equations to **solve** problems involving projectile motion.

Projectiles

- Objects that are thrown or launched into the air and are subject to gravity are called **projectiles.**
- **Projectile motion** is the curved path that an object follows when thrown, launched,or otherwise projected near the surface of Earth.
- If air resistance is disregarded, projectiles follow **parabolic trajectories.**

Projectiles, *continued*

- **Projectile motion is free fall with an initial horizontal velocity.**
- The yellow ball is given an initial horizontal velocity and the red ball is dropped. Both balls fall at the same rate.
  - *In this book, the horizontal velocity of a projectile will be considered constant.*
  - *This would not be the case if we accounted for air resistance.*

Projectile Motion

Click below to watch the Visual Concept.

Visual Concept

Kinematic Equations for Projectiles

- How can you know the displacement, velocity, and acceleration of a projectile at any point in time during its flight?
- One method is to resolve vectors into components, then apply the simpler one-dimensional forms of the equations for each component.
- Finally, you can recombine the components to determine the resultant.

End of Slide

Kinematic Equations for Projectiles, *continued*

- To solve projectile problems, apply the **kinematic equations** in the **horizontal and vertical directions.**

- In the **vertical** direction, the acceleration a_y **will equal** $-g$ ($-9.81\ m/s^2$) because the only vertical component of acceleration is free-fall acceleration.

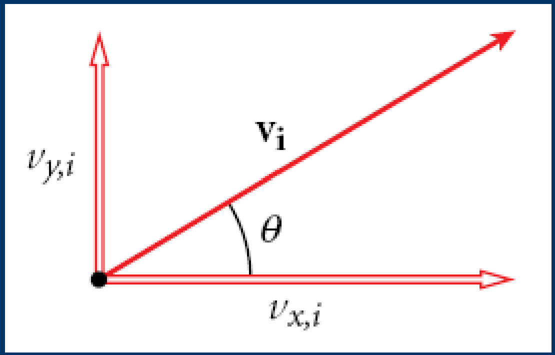
- In the **horizontal** direction, the acceleration is zero, so the **velocity is constant.**

End of Slide

Kinematic Equations for Projectiles, *continued*

- **Projectiles Launched Horizontally**
  - The initial vertical velocity is 0.
  - The initial horizontal velocity is the initial velocity.

- **Projectiles Launched At An Angle**
  - Resolve the initial velocity into *x* and *y* components.
  - The initial vertical velocity is the *y* component.
  - The initial horizontal velocity is the *x* component.



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Sample Problem

Projectiles Launched At An Angle

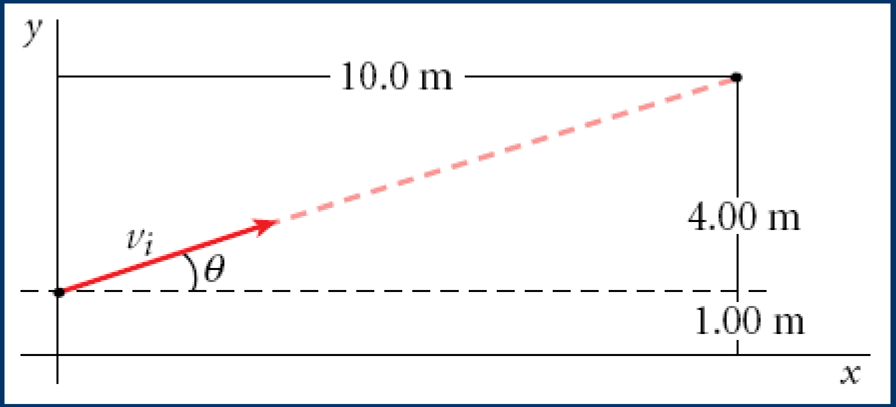
A zookeeper finds an escaped monkey hanging from a light pole. Aiming her tranquilizer gun at the monkey, she kneels 10.0 m from the light pole,which is 5.00 m high. The tip of her gun is 1.00 m above the ground. At the same moment that the monkey drops a banana, the zookeeper shoots. If the dart travels at 50.0 m/s,will the dart hit the monkey, the banana, or neither one?

End of Slide

Sample Problem, *continued*

1 . Select a coordinate system.

The positive *y*-axis points up, and the positive *x*-axis points along the ground toward the pole. Because the dart leaves the gun at a height of 1.00 m, the vertical distance is 4.00 m.



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Sample Problem, *continued*

2 . Use the inverse tangent function to find the angle that the initial velocity makes with the *x*-axis.

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{4.00\ \text{m}}{10.0\ \text{m}}\right) = 21.8^\circ$$

Sample Problem, *continued*

3 . Choose a kinematic equation to solve for time.

Rearrange the equation for motion along the *x*-axis to isolate the unknown Δt, which is the time the dart takes to travel the horizontal distance.

$$\Delta x = (v_i \cos \theta)\Delta t$$

$$\Delta t = \frac{\Delta x}{v_i \cos \theta} = \frac{10.0 \text{ m}}{(50.0 \text{ m/s})(\cos 21.8°)} = 0.215 \text{ s}$$

End of Slide

Sample Problem, *continued*

4 . Find out how far each object will fall during this time. Use the free-fall kinematic equation in both cases.

For the banana, $v_i = 0$. Thus:

$$\Delta y_b = ½a_y(\Delta t)^2 = ½(–9.81\ \text{m/s}^2)(0.215\ \text{s})^2 = –0.227\ \text{m}$$

The dart has an initial vertical component of velocity equal to $v_i \sin\theta$, so:

$$\Delta y_d = (v_i \sin\theta)(\Delta t) + ½a_y(\Delta t)^2$$

$$\Delta y_d = (50.0\ \text{m/s})(\sin 21.8°)(0.215\ \text{s}) + ½(–9.81\ \text{m/s}^2)(0.215\ \text{s})^2$$

$$\Delta y_d = 3.99\ \text{m} – 0.227\ \text{m} = 3.76\ \text{m}$$

End of Slide

Sample Problem, *continued*

5 . Analyze the results.

Find the final height of both the banana and the dart.

$$y_{banana,\,f} = y_{b,i} + \Delta y_b = 5.00 \text{ m} + (-0.227 \text{ m})$$

$$\boxed{y_{banana,\,f} = 4.77 \text{ m above the ground}}$$

$$y_{dart,\,f} = y_{d,i} + \Delta y_d = 1.00 \text{ m} + 3.76 \text{ m}$$

$$\boxed{y_{dart,\,f} = 4.76 \text{ m above the ground}}$$

The dart hits the banana. The slight difference is due to rounding.

End of Slide

Preview

- Objectives
- Frames of Reference
- Relative Velocity

Objectives

- **Describe** situations in terms of frame of reference.
- **Solve** problems involving relative velocity.

Frames of Reference

- If you are moving at **80 km/h** north and a car passes you going **90 km/h**, to you the faster car seems to be moving north at **10 km/h**.
- Someone standing on the side of the road would measure the velocity of the faster car as **90 km/h** toward the north.
- This simple example demonstrates that velocity measurements depend on the **frame of reference** of the observer.

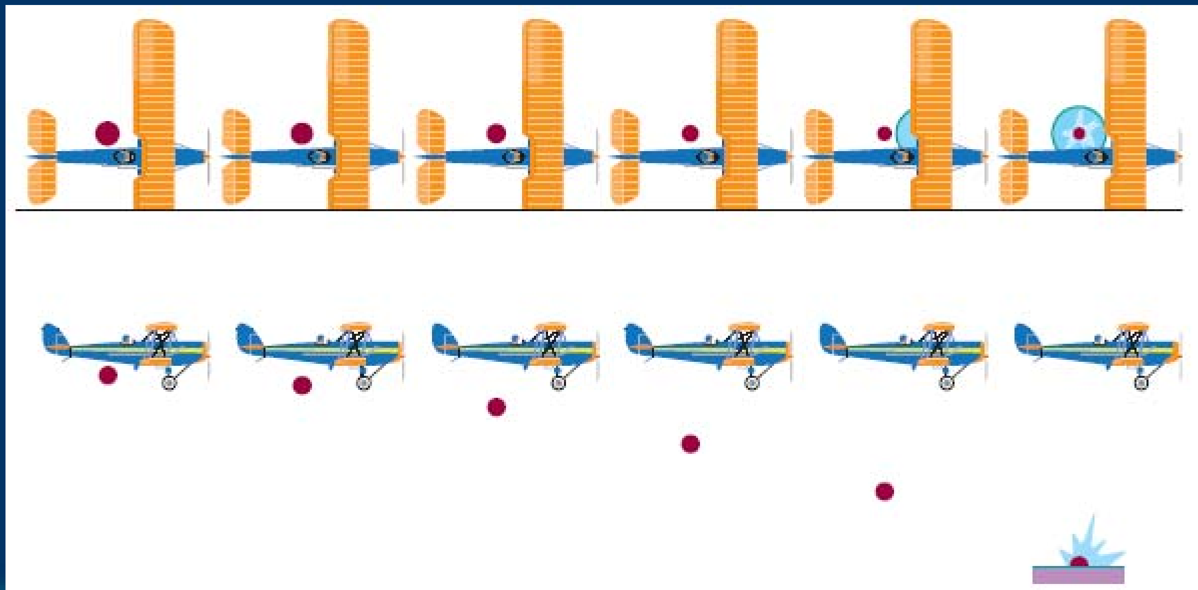
End of Slide

Frames of Reference, *continued*

Consider a stunt dummy dropped from a plane.

(a) When viewed from the plane, the stunt dummy falls straight down.

(b) When viewed from a stationary position on the ground, the stunt dummy follows a parabolic projectile path.



Relative Motion

Click below to watch the Visual Concept.

Visual Concept

Relative Velocity

- When solving relative velocity problems, write down the information in the form of velocities with subscripts.

- Using our earlier example, we have:
  - $\mathbf{v}_{se}$ = +80 km/h north (se = slower car with respect to Earth)
  - $\mathbf{v}_{fe}$ = +90 km/h north (fe = fast car with respect to Earth)
  - unknown = $\mathbf{v}_{fs}$ (fs = fast car with respect to slower car)

- Write an equation for $\mathbf{v}_{fs}$ in terms of the other velocities. The subscripts start with **f** and end with **s**. The other subscripts start with the letter that ended the preceding velocity:
  - $\mathbf{v}_{fs} = \mathbf{v}_{fe} + \mathbf{v}_{es}$

End Of Slide

Relative Velocity, *continued*

- An observer in the slow car perceives Earth as moving south at a velocity of 80 km/h while a stationary observer on the ground (Earth) views the car as moving north at a velocity of 80 km/h. In equation form:
  - $\mathbf{v}_{es} = -\mathbf{v}_{se}$
- Thus, this problem can be solved as follows:
  - $\mathbf{v}_{fs} = \mathbf{v}_{fe} + \mathbf{v}_{es} = \mathbf{v}_{fe} - \mathbf{v}_{se}$
  - $\mathbf{v}_{fs}$ = (+90 km/h n) – (+80 km/h n) = +10 km/h n
- A general form of the relative velocity equation is:
  - $\mathbf{v}_{ac} = \mathbf{v}_{ab} + \mathbf{v}_{bc}$

End of Slide

Relative Velocity

Click below to watch the Visual Concept.

Visual Concept