Ch 1-Aside: Math is Physics

I) Back ground
   A) In 1687 (Sir) Isaac Newton published ***Philosophiæ Naturalis Principia Mathematica***, Latin for "Mathematical Principles of Natural Philosophy", often referred to as simply the ***Principia***,
      1) Though not immediately accepted it quickly became accepted as the framework for a whole new scientific field, Physics.
      2) It directly addressed the mathematical relationships defining linear motion, the laws of motion, planetary motion and universal gravitation
      3) Although he did not know it, the mathematical concepts developed in the book ("infinitesimal geometry") would become a new field of mathematics (Calculus)
   B) Newton once said, "If I have seen far it is only because I have stood on the shoulders of giants."
      1) His thesis required fields that had taken centuries to develop
         (a) Arithmetic (prehistory)
         (b) Geometry (Euclid, Greece – 3rd century)
         (c) Trigonometry (Pythagoras – 3rd century)
         (d) Algebra (between 3rd and 9th century)
      2) While full understanding of these field is not necessary, some part are vital to your ability to grasp physics

II) Dimensions & Units
   A) Real-world numbers are generally measurements of some kind
      1) A <u>dimension</u> describes what quantity is being measured (distance, time)
         (a) The second dimension
      2) <u>Units</u> are the agreed standard by which we measure a dimension
      3) When quantities have different dimensions they CANNOT be added/subtracted, but CAN be multiplied/divided
   B) Ancient Units
      1) Cubits – elbow to tip of middle finger
   C) Royal System
      1) Explanation – same units, use the king as standard.
   D) Metric System
      1) Standardization and decimals already considered and discussed by scientists and officials throughout Europe
      2) The French Revolution
      3) The spread of Metrics
         (a) Napoleon
         (b) Jefferson
         (c) The 1800s saw countries settling governance questions and a widespread turn towards metrics

4) The SI

E) US Customary

1) Origins – British Empirical System

2) History

(a) Metrification – the act of converting measurements in use from traditional to metric

(b) Resistance:

- In the late 1800s, while other nations tried to leave their royal roots behind the US saw resistance to such a move by several organizations including:

(c) "The International Institute for Preserving and Perfecting Weights and Measures in the late 19th century. Advocates of the customary system saw the French Revolutionary, or metric, system as atheistic.[6] An auxiliary of the Institute in Ohio published a poem reading:

"Then down with every "metric" scheme
        Taught by the foreign school,
We'll worship still our Father's God!
        And keep our Father's "rule"!
A perfect inch, a perfect pint,
        The Anglo's honest pound,
Shall hold their place upon the earth,
        Till time's last trump shall sound![6]"
        -Wikipedia

3) Current

(a) In 1989 Congress passed the Omnibus Trade and Competitiveness Act of 1988 the purpose of which is, in part, to support the SI system as "the preferred system of weights and measures for U.S. trade and commerce".

III) Math Tools

A) Physicists use **equations** to describe measured or predicted relationships between physical quantities.

B) **Variables** and other specific quantities are abbreviated with letters that are **boldfaced** or *italicized*.

C) **Units** are abbreviated with regular letters, sometimes called roman letters.

1) Many symbols are used in science:

(a) α (alpha) – proportionality

- This like an equal sign that needs a constant

(b) Δ (delta) – the difference ('change in')

- Calculate by subtracting starting value from final value
- i.e. $\Delta x = x_{final} - x_{initial}$

(c) Σ (Sigma) – T sum of… (add all values)

D) Graphs

1) visual representations of the relationship
2) Not always clearly presenting one equation or another
3) TYPES OF GRAPHS (linear v. exponential v. logarithmic)

E) Fractions

1) A convenient way of denoting division
2) Because it is a combination of division and parentheses the order of operations is important (i.e. (1+2)/(3-x) is $\frac{1+2}{3-x}$
3) A <u>rational equation</u> is an equation where at least one denominator contains a variable.
   (a) When a denominator contains a variable, there is usually a restriction on the domain, at least theoretically (The variable cannot take on any number that would cause any denominator to be zero.)
   (b) The first step is solving a rational equation is to **<u>convert the equation to an equation without denominators</u>**.
   (c) Next solve for your variable

F) Geometric proofs

1) A method for writing and following mathematical explanations
2) Draw a line down the paper-One side for work, the other for explanation
3) Write one step at a time & explain it

IV) Arithmetic

A) Identities

1) <u>Identity</u> – a math term for something that is always true
   - (Number) x 1 = number
   - (Number) / 1 = number
2) An <u>inverse</u> is an opposite. They may be additive (-x) or multiplicative ($^1/_x$)
   - -(2) = -2
   - 2^{-1}=1/2

B) Order of Operations

1) ALL of math is simply an expression of logic:
   - Putting two groups of four together gives 8 so 2+2=8
2) Because the notations have meaning and logic, there is an order which makes sense:
   (a) Parentheses exist to define what comes first : (2+1)*2 MEANS add 2 and 1 before multiplying
3) Multiplication (also called <u>*scaling*</u>) is shorthand for repeated addition
   - 2 x 3 means 2+2+2 or 3+3
4) Exponents (exponentiation) corresponds to repeated multiplication
   - 2^3 means 2x2x2
     - ⇒ A negative exponent means repeated multiplication of (multiplicative) ***inverse***

- $2^{-3} = (\frac{1}{2})^3 = (\frac{1}{2})\times(\frac{1}{2})\times(\frac{1}{2})$
- i.e. x^{-2} means $\frac{1}{x} \times \frac{1}{x}$

(b) e.g. 3×10^{-2} becomes $(3) \times (\frac{1}{10})^2$ becomes $(3) \times (\frac{1}{100})$ becomes $\frac{3}{100}$ becomes .03

5) Therefore: Parentheses → Exponents → Multiplications → addition

C) Fractions

1) A convenient way of denoting division
2) Because it is a combination of division and parentheses the order of operations is important
   - (i.e. (1+2)/(3-x) is $\frac{1+2}{3-x}$)

V) Scientific notation

A) Multiple notation systems exist for describing numbers (fractions, decimal, etc).

1) Notation should fit needs (i.e. $^3/_7$ is easier to multiply than $0.\overline{42857}$)

B) Scientific Notation is a way of writing numbers using exponents to show how large or small a number is in terms of powers of ten.

1) Relative size is called Scale (2; LP)
   (a) <u>Order of Magnitude</u> – the power of 10
      ⇒ (10^2 has an order of magnitude of 2)
2) Scientist work over a large range of scales, and scientific notation makes scale easy to work with:
   (a) Consider 1×10^{24} vs. 1×10^{-24}, or : 1,000,000,000,000,000,000,000,000 vs. 0.000 000 000 000 000 000 000 001

C) Form:

1) $n \times 10^m$ where n is a real number { $1 \leq n < 10$ } and m is an integer {whole number}
2) This form is standardized-there is only ONE correct way to right a number
3) The *mantissa* (n) must have one digit before the decimal so $.9 \times 10^3$ is NOT okay, nor is 11×10^3

D) Translation

1) The '$\times10^m$' can be thought of as a separate factor, so translating between standard and scientific notation should be logical
2) Standard notation into Scientific notation:
   (a) 200 can be written as (2) x (100)
   (b) 100 can be written as 10^2
   (c) So: 200 → 2×10^2
3) Scientific notation into Standard notation:
   (a) 2×10^3 can be written as (2) x (1000) or 2000

E) Notation:

1) The powers of ten may be written using the "carrot" symbol to indicate the exponent of the power of ten (1 x 10^3.)
2) On calculators the button EE, or just E may represent " x10^ " ($1x10^3$ is entered [1] [EE] [3])

F) In calculations (theory explained below: exponents)
1) Adding and Subtracting
(a) Both numbers must be have the same order of magnitude
(b) Proceed as normal (order of magnitude does not change)
2) Multiplying and dividing
(a) Treat *mantissa* and order of magnitude (x10^_) separately
(b) Recall that multiplying two numbers (same base-10) is a matter of adding exponents
⇒ $x \bullet x = x^2$ because 1 + 1 is 2
⇒ $2x10^2 \bullet 3x10^3 = 6x10^5$ because 2 + 3 is 5
⇒ $1x10^{-2} \bullet 2x10^3 = 2x10^1$ because -2 + 3 is 1
(c) Dividing is a matter of subtraction
⇒ x / x = 1 because 1 - 1 is 0 (and anything to the 0 is one)
⇒ $8x10^2 / 4x10^3 = 2x10^{-1}$ because 2 - 3 is -1
⇒ $9x10^{-2} / 3x10^3 = 10^{-5}$ because -2 - 3 is -5

G) Using in estimation
1) An <u>order of magnitude</u> estimation is a quick efficient way to set expectations
2) for a given operation determine the orders of magnitude for each number, ignore the mantissa and perform the calculation
⇒ 2,100 x 35,000 becomes $10^3 \times 10^4 = 10^7$
⇒ So you estimate your answer to be about 10,000,000 (actual= ________)
3) When adding/subtracting, safely ignore the smaller number
• $1x10^0 - 1x10^{-9} = 0.9999999999 \sim 1x10^0$

Simple powers of ten
1 x 10^9 = 1,000,000,000
1 x 10^6 = 1,000,000
1 x 10^3 = 1,000
1 x 10^1 = 10
1 x 10^0 = 1
1 x 10^-1 = 1 / 10
1 x 10^-3 = 1 / 1000
1 x 10^-6 = 1 / 1,000,000
1 x 10^-9 = 1 / 1,000,000,000

VI) Significant Figures
A) Concept clarity
1) To be <u>accurate</u> is to be right
(a) being close to a target
(b) % error
2) To be <u>precise</u> is to be perfectly clear
(a) Hitting the same place on a dart board over and over is precision throwing
(b) precision is important when it comes to making measurements

B) Exist to communicate how well a number was measured:

1) 1.0 is not as precise as 1.00
   (a) This tells scientists how many digits were measure
2) There are rules that govern how to use numbers with SigFigs
   (a) The upshot is that no calculated number is more precise than the least precise input, so the number of SigFigs used will show the least precise measurement

VII)

VIII) Algebra

A) You must always do the same thing to both sides of an equation, regardless of what operation you are applying.
   1) Follow the reverse of the mnemonic: "Please excuse my dear aunt Sally"
   2) Start with the variable and work your way out
      - $(4x^2-2)-1=1$ Looking at the x, the ***last thing*** to do is parentheses, the first is subtraction
      - $(4x^2-2) = 2$ Now, within the parentheses do reverse order of operations-subtraction
      - $4x^2 = 4$ Do mult./div. before exponents
      - $x^2 = 1$ Simplify exponents
      - $x = 1$
   3) A <u>rational equation</u> is an equation where at least one denominator contains a variable.
      (a) The first step is solving a rational equation is to **<u>convert the equation to an equation without variables in the denominators</u>**.
      (b) Next solve for your variable
      - To begin consider the ratio and proportion equation:
      - $\boldsymbol{a}/\boldsymbol{b} = \boldsymbol{c}/\boldsymbol{d} + \boldsymbol{e}.$ Mult. by d
      - $\boldsymbol{ad}/\boldsymbol{b} = \boldsymbol{c} + \boldsymbol{ed}$ Multiply by b
      - $\boldsymbol{ad} = \boldsymbol{cb} + \boldsymbol{bed}$ solve normally
   4) <u>Complex fractions</u> feature fractions in fractions
      (a) maintain the notion that they are all simple arithmetic and parentheses in short-hand
      (b) Solve: $\frac{\frac{3}{2x^2}}{\frac{2}{7x}} = 1$
      - $(\frac{3}{2x^2})/(\frac{2}{7x}) = 1$ re-write

(c) $\left(\frac{2x^2}{2x^2}\right)[(\frac{3}{2x^2})/(\frac{2}{7x})](\frac{7x}{7x})$ = 1 find factors for simplifying (*Note: I am only using identities (x1) so I'm not changing anything)

(d) $\left(\frac{\cancel{2x^2}}{2x^2}\right)\frac{\frac{3}{\cancel{2x^2}}}{\frac{2}{\cancel{7x}}}(\frac{7x}{\cancel{7x}})$ = $\frac{(3)(7x)}{(2)(2x^2)}$=$\frac{21x}{4x^2}$ = 1 re-write and simplify

(e) Now that I'm down to a basic fraction I just use algebra

(f) So: $\frac{21x}{4x^2}$ = 1

(g) becomes: $21x = 4x^2$

(h) becomes: $21 = 4x$

(i) so: x = 5.25

IX) Significant Figures

A) Concept clarity

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X) Unit Conversions

A) Theory:

1) as long as you multiply a number by one you don't change it.
2) any number divided by itself is one
3) if you put the two sides of an equation into a fraction, you can use that as a one
   - 1 ft = 12in can be used as $\left(\frac{1ft}{12in}\right) = 1 = (\frac{12in}{1ft})$

B) Method

1) Equation – write what unit you want set equal to the value you have:
   - __ft = 36in
2) Conversion Factors – use equivalent factor to cancel units until you have what you want
   - __ft = 36in $\left(\frac{1ft}{12in}\right) =$
3) Simplification

- __ft = 36~~in~~ $\left(\frac{1ft}{12\cancel{in}}\right)$ =3ft

4) Do this as many times as needed

XI)