Notes on Numbers

I) A brief history of Physics

A) **Aristotle, the foremost Greek scientist, studied motion and divided it into two types: *natural motion* and *violent motion*.**

1) During Aristotle's time, "natural motion" on Earth was thought to be either straight up or straight down: It was "natural" for heavy things to fall and for very light things to rise.
2) "Violent motion" was *imposed* motion and it was the result of forces that pushed or pulled.
3) The "proper state" of all objects is one of rest.

B) **Copernicus reasoned that the simplest way to interpret astronomical observations was to assume that Earth and the other planets move around the sun.**

1) Copernicus' idea of motion in space was extremely controversial at the time, because most people believed that Earth was at the center of the universe.
2) Copernicus worked on his ideas in secret to escape persecution. At the urging of his close friends, he published his ideas.

C) **Galileo argued that only when friction is present—as it usually is—is a force needed to keep an object moving.**

1) One of Galileo's greatest contributions to physics was demolishing the notion that a force is necessary to keep an object moving.
2) **Friction** is the force that acts between materials that touch as they move past each other.
3) Thought experiment:
   (a) If a ball rolling down a hill gets faster and…
   (b) a ball rolling up a hill gets slower…
   (c) a ball rolling on a smooth horizontal plane has almost constant velocity, and if friction were entirely absent, the ball would move forever.
4) Galileo also stated that the tendency of a moving body to keep moving is natural and that every object resists change to its state of motion.
   (a) The property of a body to resist changes to its state of motion is called **inertia.**

D) Newton

1) The week Galileo died Newton was born
2) While working as a professor he submitted a paper to win a cash prize from the Royal Society
   (a) Called the *Principia Mathematica* it propelled Newton into becoming the father of both Modern Physics and Calculus

II) Scientific notation

A) Multiple notation systems exist for describing numbers (fractions, decimal, etc).

1) Notation should fit needs (i.e. ${}^{3}/_{7}$ is easier to multiply than $0.\overline{42857}$)

B) Scientific Notation is a way of writing numbers using exponents to show how large or small a number is in terms of powers of ten.

1) Relative size is called Scale (2; LP)

(a) Order of Magnitude – the power of 10

(10^2 has an order of magnitude of 2)

2) Scientist work over a large range of scales, and scientific notation makes scale easy to work with:

(a) Consider $1\text{x}10^{24}$ vs. $1\text{x}10^{-24}$, or : 1,000,000,000,000,000,000,000,000 vs. 0.000 000 000 000 000 000 000 001

C) Form:

$n \times 10^m$ where n is a real number { $1 \leq n < 10$ } and m is an integer

1) This form is standardized-there is only ONE correct way to right a number

2) The *mantissa* (n) must have one digit before the decimal so $.9 \times 10^3$ is NOT okay, nor is 11×10^3

Simple powers of ten

1 x 10^9 = 1,000,000,000
1 x 10^6 = 1,000,000
1 x 10^3 = 1,000
1 x 10^1 = 10
1 x 10^0 = 1
1 x 10^-1 = 1 / 10
1 x 10^-3 = 1 / 1000
1 x 10^-6 = 1 / 1,000,000
1 x 10^-9 = 1 / 1,000,000,000

D) Translation

1) The 'x10^m' can be thought of as a separate factor, so translating between standard and scientific notation should be logical

2) Standard notation into Scientific notation:

(a) 200 can be written as (2) x (100)

(b) 100 can be written as 10^2

(c) So: 200 → 2 x 10^2

3) Scientific notation into Standard notation:

(a) 2 x 10^3 can be written as (2) x (1000) or 2000

E) Notation:

1) The powers of ten may be written using the "carrot" symbol to indicate the exponent of the power of ten (1 x 10^3.)

2) On calculators the button EE, or just E may represent " x10^ " ($1\text{x}10^3$ is entered [1] [EE] [3])

F) In calculations (theory explained below: exponents)

1) Adding and Subtracting
   (a) Both numbers must be have the same order of magnitude
   (b) Proceed as normal (order of magnitude does not change)
2) Multiplying and dividing
   (a) Treat number and order of magnitude (x10^_) separately
   (b) Recall that multiplying two numbers (same base-10) is a matter of adding exponents
      - ⇒ $x \bullet x = x^2$ because 1 + 1 is 2
      - ⇒ $2x10^2 \bullet 3x10^3 = 6xy10^5$ because 2 + 3 is 5
      - ⇒ $1x10^{-2} \bullet 2x10^3 = 2x10^1$ because -2 + 3 is 1
   (c) Dividing is a matter of subtraction
      - ⇒ x / x = 1 because 1 - 1 is 0 (and anything to the 0 is one)
      - ⇒ $8x10^2 / 4x10^3 = 2x10^{-1}$ because 2 - 3 is -1
      - ⇒ $9x10^{-2} / 3x10^3 = 10^{-5}$ because -2 - 3 is -5

G) Using in estimation
1) An <u>order of magnitude</u> estimation is a quick efficient way to set expectations
2) for a given operation determine the orders of magnitude for each number, ignore the mantissa and perform the calculation
   - ⇒ 2,100 x 35,000 becomes $10^3 \times 10^4 = 10^7$
   - ⇒ So you estimate your answer to be about 10,000,000 (actual= ________)
3) When adding/subtracting, safely ignore the smaller number
   - $1x10^0 - 1x10^{-9} = 0.9999999999 \sim 1x10^0$

III) Dimensions & Units

A) Real-world numbers are generally measurements of some kind
1) A <u>dimension</u> describes what quantity is being measured (distance, time)
   (a) The second dimension
2) <u>Units</u> are the agreed standard by which we measure a dimension

B) Ancient Units
1) Cubits – elbow to tip of middle finger

C) Royal System (Middle Ages)
1) Generally used the king as standard. (foot, inch)

D) Metric System
1) Prominent thinkers thought standardization and decimals should be adopted by scientists and officials throughout Europe
2) The French Revolution – Overthrow the monarchy
3) The spread of Metrics
   (a) Napolean
   (b) Jefferson

(c) The 1800s saw countries settling governance questions and a widespread turn towards metrics

4) The Standard International system grew from metrics, and is used almost universally

E) US Customary

1) Origins-generally these units were British units before the empire standardized in 1824

2) History

(a) Metrification – the act of converting measurements in use from traditional to metric

(b) Resistance:

- In the late 1800s, while other nations tried to leave their royal roots behind the US saw resistance to such a move by several organizations including:

"The International Institute for Preserving and Perfecting Weights and Measures in the late 19th century. Advocates of the customary system saw the French Revolutionary, or metric, system as atheistic.[6] An auxiliary of the Institute in Ohio published a poem reading:

"Then down with every "metric" scheme
    Taught by the foreign school,
We'll worship still our Father's God!
    And keep our Father's "rule"!
A perfect inch, a perfect pint,
    The Anglo's honest pound,
Shall hold their place upon the earth,
    Till time's last trump shall sound![6]"
    -Wikipedia

3) Current

(a) In 1989 Congress passed the Omnibus Trade and Competitiveness Act of 1988 the purpose of which is, in part, to support the SI system as "the preferred system of weights and measures for U.S. trade and commerce".

I) Arithmetic

A) Basic Operations

1) Multiplication (also called *scaling*) is shorthand for repeated addition

- 2 x 3 means 2+2+2 or 3+3

B) Exponents (exponentiation) corresponds to repeated multiplication

- 2^3 means 2x2x2

(An inverse is an opposite. They may be additive (-x) or multiplicative ($^1/_x$))

1) A negative exponent means repeated multiplication of (multiplicative) ***inverse***
   (a) i.e. x^{-2} means $\frac{1}{x} \times \frac{1}{x}$
   (b) e.g. 3×10^{-2} becomes $(3) \times (\frac{1}{10^2})$ becomes $(3) \times (\frac{1}{100})$ becomes $\frac{3}{100}$ becomes .03

II) Algebra

A) You must always do the same thing to both sides of an equation, regardless of what operation you are applying.

1) Follow the reverse of the mnemonic: "Please excuse my dear aunt Sally"
2) Start with the variable and work your way outside - in

Solve: $2(4x^2-2)-1 = 3$

- $2(4x^2-2)-1 = 3$ Looking at the x, the ***last thing*** to do is parentheses, the first is subtraction (ADD 1)
- $2(4x^2-2) = 4$ Again, save the parentheses, divide by 2
- $4x^2-2 = 2$ Now, do reverse order of operations (Add 1)
- $4x^2 = 4$ Do mult./div. before exponents (/4)
- $x^2 = 1$ Simplify exponents (square root)
- $x = 1$

3) A *rational equation* is an equation where at least one denominator contains a variable.
   (a) The first step is solving a rational equation is to **<u>convert the equation to an equation without variables in the denominators</u>**.
   (b) Next solve for your variable
   - To begin consider the ratio and proportion equation:
   - $\boldsymbol{a}/\boldsymbol{b} = \boldsymbol{c}/\boldsymbol{d}.$ Mult. by d
   - $\boldsymbol{ad}/\boldsymbol{b} = \boldsymbol{c}.$ Multiply by b
   - $\boldsymbol{ad} = \boldsymbol{cb}.$ solve normally

Consider:

- $\boldsymbol{a}/\boldsymbol{b} = \boldsymbol{c}/\boldsymbol{d} + \boldsymbol{e}.$ Mult. by d
- $\boldsymbol{d} * \left(\boldsymbol{a}/\boldsymbol{b}\right) = \left(\boldsymbol{c}/\boldsymbol{d} + \boldsymbol{e}\right) * \boldsymbol{d}$
- $\boldsymbol{ad}/\boldsymbol{b} = \boldsymbol{c} + \boldsymbol{e}$ →**WRONG!**

- $ad/_{b} = c + ed$ - Multiply by b

- $ad = cb + bed$ solve normally

4) Complex fractions feature fractions in fractions
   (a) maintain the notion that they are all simple arithmetic and parentheses in short-hand
   (b) Solve:
   - $\frac{\frac{3}{2x^2}}{\frac{2}{7x}} = 1$

     $\Rightarrow$ $(\frac{3}{2x^2})/(\frac{2}{7x})$ =1 re-write

   - $\left(\frac{2x^2}{2x^2}\right)[(\frac{3}{2x^2})/(\frac{2}{7x})](\frac{7x}{7x})$ = 1 find factors for simplifying (*Note: I am only using identities (x1) so I'm not changing anything)

   - $\left(\frac{\cancel{2x^2}}{2x^2}\right)\frac{\frac{3}{\cancel{2x^2}}}{\frac{2}{\cancel{7x}}}(\frac{7x}{\cancel{7x}})$ = $\frac{(3)(7x)}{(2)(2x^2)}$=$\frac{21x}{4x^2}$ = 1 re-write and simplify
   - Now that I'm down to a basic fraction I just use algebra
   - So: $\frac{21x}{4x^2}$ = 1
   - becomes: $21x = 4x^2$
   - becomes: $21 = 4x$
   - so: x = 5.25

III) Significant Figures

A) Concept clarity

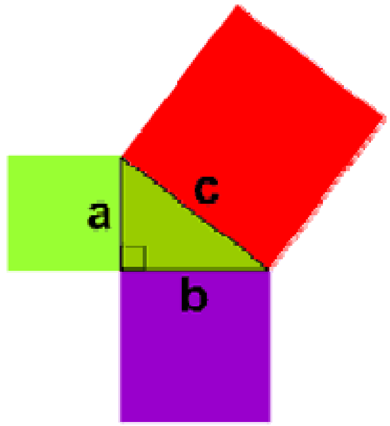
1) To be accurate is to be right
   (a) being close to a target
   (b) % error
2) To be precise is to be perfectly clear

        (a) Hitting the same place on a dart board over and over is precision throwing
        (b) precision is important when it comes to making measurements
    B) Exist to communicate how well a number was measured:
        1) 1.0 is not as precise as 1.00
            (a) This tells scientists how many digits were measure
        2) There are rules that govern how to use numbers with SigFigs
            (a) The upshot is that no calculated number is more precise than the least precise input, so the number of SigFigs used will show the least precise measurement

IV) Unit Conversions
    A) Theory:
        1) as long as you multiply a number by one you don't change it.
        2) any number divided by itself is one
        3) if you put the two sides of an equation into a fraction, you can use that as a one
            - 1 ft = 12in can be used as $\left(\frac{1ft}{12in}\right) = 1 = (\frac{12in}{1ft})$
    B) Method
        1) Equation – write what unit you want set equal to the value you have:
            - __ft = 36in
        2) Conversion Factors – use equivalent factor to cancel units until you have what you want
            - __ft = 36in $\left(\frac{1ft}{12in}\right) =$
        3) Simplification
            - __ft = 36~~in~~ $\left(\frac{1ft}{12\cancel{in}}\right)$ =3ft
        4) Do this as many times as needed inserting extra conversion factors into your equation:

V) Trigonometry - The math of triangles
    A) Pythagorean theorem – allows the length of one side of a right triangle to be found if the other legs are known:

$$a^2 + b^2 = c^2$$



B) Sine functions - used to calculate the length of a side when one length and one angle is known

1) **Summary: SOHCAHTOA**

(a) Sine of the angle = Opposite side / Hypotenuse

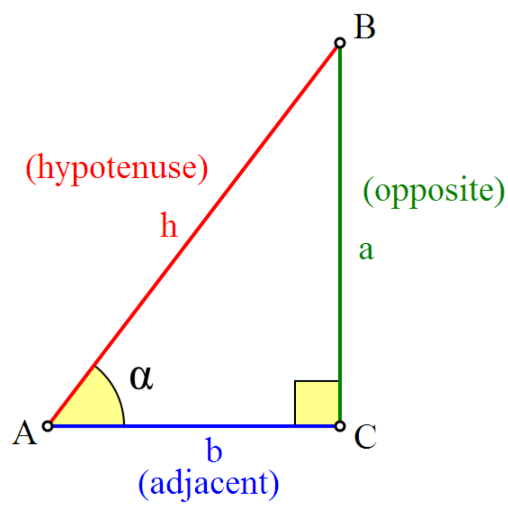
- **Sin θ = OPP / HYP**

(b) Cosine of the angle = Adjacent side / Hypotenuse

- **Cos θ = ADJ / HYP**

(c) Tangent of the angle = Opposite side / Adjacent side

- **Tan θ = OPP / ADJ**



2) **Inverse functions – allows you to use two lengths to calculate an angle:**

- **Sin^{-1} (OPP / HYP) = θ**
- **Cos^{-1} (ADJ / HYP) = θ**
- **Tan^{-1} (OPP / ADJ) = θ**