NAME ______________________ DATE ______________ CLASS ______________

Motion in One Dimension

# Problem A

## AVERAGE VELOCITY AND DISPLACEMENT

**PROBLEM**

**To qualify for the finals in a racing event, a race car must achieve an average speed of $2.50 \times 10^2$ km/h on a track with a total length of 1.60 km. If a particular car covers the first half of the track at an average speed of $2.30 \times 10^2$ km/h, what minimum average speed must it have in the second half of the event to qualify?**

**SOLUTION**

**Given:**

$v_{tot,\,avg} = 2.50 \times 10^2$ km/h

$v_{1,\,avg} = 2.30 \times 10^2$ km/h

$\Delta x_{tot} = 1.60$ km

$\Delta x_1 = \Delta x_2 = \frac{1}{2}\Delta x_{tot}$

**Unknown:** $v_{2,\,avg} = ?$

Use the definition of average velocity, and rearrange it to solve for time.

$$v_{avg} = \frac{\Delta x}{\Delta t}$$

$$\Delta t_1 = \text{time required to travel } \Delta x_1 = \frac{\Delta x_1}{v_{1,\,avg}}$$

$$\Delta t_2 = \text{time required to travel } \Delta x_2 = \frac{\Delta x_2}{v_{2,\,avg}}$$

$$\Delta t_{tot} = \Delta t_1 + \Delta t_2 = \frac{\Delta x_1}{v_{1,\,avg}} + \frac{\Delta x_2}{v_{2,\,avg}} = \frac{\Delta x_{tot}}{v_{tot,\,avg}}$$

Use the last two equations for $\Delta t_{tot}$ to solve for $v_{2,\,avg}$.

$$\frac{\Delta x_{tot}}{v_{tot,\,avg}} = \frac{\Delta x_1}{v_{1,\,avg}} + \frac{\Delta x_2}{vt_{2,\,avg}} = \frac{1}{2}\Delta x_{tot}\left(\frac{1}{v_{1,\,avg}} + \frac{1}{v_{2,\,avg}}\right)$$

Divide by $\frac{1}{2}\Delta x_{tot}$ on each side.

$$\frac{2}{v_{tot,\,avg}} = \frac{1}{v_{1,\,avg}} + \frac{1}{v_{2,\,avg}}$$

Rearrange the equation to calculate $v_{2,\,avg}$.

$$\frac{1}{v_{2,\,avg}} = \frac{2}{v_{tot,\,avg}} - \frac{1}{v_{1,\,avg}}$$

Invert the equation.

$$v_{2,\,avg} = \frac{1}{\left(\frac{2}{v_{tot,\,avg}} - \frac{1}{v_{1,\,avg}}\right)}$$

$$v_{2,\,avg} = \frac{1}{\left(\frac{2}{2.50 \times 10^2 \text{ km/h}}\right) - \left(\frac{2}{2.30 \times 10^2 \text{ km/h}}\right)}$$

$$v_{2,\,avg} = \frac{1}{8.00 \times 10^{-3}\ \text{h/km} - 4.35 \times 10^{-3}\ \text{h/km}}$$

$$v_{2,\,avg} = \frac{1\ \text{km}}{3.65 \times 10^{-3}\ \text{h}}$$

$$v_{2,\,avg} = \boxed{274\ \text{km/h}}$$

## ADDITIONAL PRACTICE

**1.** The fastest helicopter, the Westland Lynx, can travel 3.33 km in the forward direction in just 30.0 s. What is the average velocity of this helicopter? Express your answer in both meters per second and kilometers per hour.

**2.** The fastest airplane is the Lockheed SR-71 *Blackbird*, a high-altitude spy plane first built in 1964. If an SR-71 is clocked traveling 15.0 km west in 15.3 s, what is its average velocity in kilometers per hour?

**3.** At its maximum speed, a typical snail moves about 4.0 m in 5.0 min. What is the average speed of the snail?

**4.** The arctic tern migrates farther than any other bird. Each year, the Arctic tern travels $3.20 \times 10^4$ km between the Arctic Ocean and the continent of Antarctica. Most of the migration takes place within two four-month periods each year. If a tern travels $3.20 \times 10^4$ km south in 122 days, what is its average velocity in kilometers per day?

**5.** Suppose the tern travels $1.70 \times 10^4$ km south, only to encounter bad weather. Instead of trying to fly around the storm, the tern turns around and travels $6.00 \times 10^2$ north to wait out the storm. It then turns around again immediately and flies $1.44 \times 10^4$ km south to Antarctica. What are the tern's average speed and velocity if it makes this trip in 122 days?

**6.** Eustace drives 20.0 km to the east when he realizes he left his wallet at home. He drives 20.0 km west to his house, takes 5.0 min to find his wallet, then leaves again. Eustace is 40.0 km east of his house exactly 60.0 min after he left the first time.

**a.** What is his average velocity?
**b.** What is his average speed?

**7.** Emily takes a trip, driving with a constant velocity of 89.5 km/h to the north except for a 22.0 min rest stop. If Emily's average velocity is 77.8 km/h to the north, how long does the trip take?

**8.** Laura is skydiving when at a certain altitude she opens her parachute and drifts toward the ground with a constant velocity of 6.50 m/s, straight down. What is Laura's displacement if it takes her 34.0 s to reach the ground?

**9.** A tortoise can run with a speed of 10.0 cm/s, and a hare can run exactly 20 times as fast. In a race, they both start at the same time, but the hare stops to rest for 2.00 min. The tortoise wins by 20.0 cm. How long does the race take?

**10.** What is the length of the race in problem 9?

# Motion In One Dimension

## Additional Practice A

| *Givens* | *Solutions* |
|---|---|
| **1.** $\Delta x = 3.33$ km forward<br>$\Delta t = 30.0$ s | $v_{avg} = \dfrac{\Delta x}{\Delta t} = \dfrac{3.33 \times 10^3 \text{ m}}{30.0 \text{ s}} = \boxed{111 \text{ m/s forward}}$<br>$v_{avg} = (111 \text{ m/s})(3600 \text{ s/h})(10^{-3} \text{ km/m}) = \boxed{4.00 \times 10^2 \text{ km/h forward}}$ |
| **2.** $\Delta x = 15.0$ km west<br>$\Delta t = 15.3$ s | $v_{avg} = \dfrac{\Delta x}{\Delta t} = \dfrac{15.0 \text{ km}}{(15.3 \text{ s})\left(\dfrac{1 \text{ h}}{3600 \text{ s}}\right)} = \dfrac{15.0 \text{ km}}{4.25 \times 10^{-3} \text{ h}} = \boxed{3.53 \times 10^3 \text{ km/h west}}$ |
| **3.** $\Delta x = 4.0$ m<br>$\Delta t = 5.0$ min | $v_{avg} = \dfrac{\Delta x}{\Delta t} = \dfrac{4.0 \text{ m}}{(5.0 \text{ min})\left(\dfrac{1 \text{ h}}{60 \text{ min}}\right)} = \boxed{48 \text{ m/h}}$ |
| **4.** $\Delta x = 3.20 \times 10^4$ km south<br>$\Delta t = 122$ days | $v_{avg} = \dfrac{\Delta x}{\Delta t} = \dfrac{3.20 \times 10^4 \text{ km}}{122 \text{ days}} = \boxed{262 \text{ km/day south}}$ |

**5.** $\Delta x_1 = 1.70 \times 10^4$ km south $= +1.70 \times 10^4$ km

$\Delta x_2 = 6.0 \times 10^2$ km north $= -6.0 \times 10^2$ km

$\Delta x_3 = 1.44 \times 10^4$ km south $= +1.44 \times 10^4$ km

$\Delta t = 122$ days

$d$ = total distance traveled = magnitude $\Delta x_1$ + magnitude $\Delta x_2$ + magnitude $\Delta x_3$

$d = 1.70 \times 10^4 \text{ km} + 6.0 \times 10^2 \text{ km} + 1.44 \times 10^4 \text{ km}$

$d = (1.70 + 0.060 + 1.44) \times 10^4 \text{ km}$

$d = 3.20 \times 10^4 \text{ km}$

$$\text{average speed} = \frac{d}{\Delta t} = \frac{3.20 \times 10^4 \text{ km}}{122 \text{ days}} = \boxed{262 \text{ km/day}}$$

$$v_{avg} = \frac{\Delta x_{tot}}{\Delta t} = \frac{\Delta x_1 + \Delta x_2 + \Delta x_3}{\Delta t}$$

$$v_{avg} = \frac{(1.70 \times 10^4 \text{ km}) + (-6.0 \times 10^2 \text{ km}) + (1.44 \times 10^4 \text{ km})}{122 \text{ days}}$$

$$v_{avg} = \frac{(1.70 - 0.060 + 1.44) \times 10^4 \text{ km}}{122 \text{ days}}$$

$$v_{avg} = \frac{3.08 \times 10^4 \text{ km}}{122 \text{ days}} = +252 \text{ km/day} = \boxed{252 \text{ km/day south}}$$

**6.** $\Delta x_1 = 20.0$ km east $= +20.0$ km

$\Delta x_2 = 20.0$ km west $= -20.0$ km

$\Delta x_3 = 0$ km

$\Delta x_4 = 40.0$ km east $= +40.0$ km

$\Delta t = 60.0$ min

**a.** $$v_{avg} = \frac{\Delta x_{tot}}{\Delta_t} = \frac{\Delta x_1 + \Delta x_2 + \Delta x_3 + \Delta x_4}{\Delta t}$$

$$v_{avg} = \frac{(20.0 \text{ km}) + (-20.0 \text{ km}) + (0 \text{ km}) + (40.0 \text{ km})}{60.0 \text{ min}}$$

$$v_{avg} = \frac{40.0 \text{ km}}{(60.0 \text{ min})\left(\dfrac{1 \text{ h}}{60 \text{ min}}\right)} = +40.0 \text{ km/h} = \boxed{40.0 \text{ km/h east}}$$

*Givens* | *Solutions*

**b.** $d$ = total distance traveled

$d$ = magnitude $\Delta x_1$ + magnitude $\Delta x_2$ + magnitude $\Delta x_3$ + magnitude $\Delta x_4$

$d = 20.0 \text{ km} + 20.0 \text{ km} + 0 \text{ km} + 40.0 \text{ km} = 80.0 \text{ km}$

$$\text{average speed} = \frac{d}{\Delta t} = \frac{80.0 \text{ km}}{(60.0 \text{ min})\left(\frac{1 \text{ h}}{60 \text{ min}}\right)} = \boxed{80.0 \text{ km/h}}$$

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**7.** $v = 89.5$ km/h north

$v_{avg} = 77.8$ km/h north

$\Delta t_{rest} = 22.0$ min

$\Delta x = v_{avg}\,\Delta t = v(\Delta t - \Delta t_{rest})$

$\Delta t(v_{avg} - v) = -v\Delta t_{rest}$

$$\Delta t = \frac{v\Delta t_{rest}}{v - v_{avg}} = \frac{(89.5 \text{ km/h})(22.0 \text{ min})\left(\frac{1 \text{ h}}{60 \text{ min}}\right)}{89.5 \text{ km/h} - 77.8 \text{ km/h}} = \frac{(89.5 \text{ km/h})(22.0 \text{ min})\left(\frac{1 \text{ h}}{60 \text{ min}}\right)}{11.7 \text{ km/h}}$$

$\Delta t = \boxed{2.80 \text{ h} = 2 \text{ h, } 48 \text{ min}}$

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**8.** $v = 6.50$ m/s downward $= -6.50$ m/s

$\Delta t = 34.0$ s

$\Delta x = v\Delta t = (-6.50 \text{ m/s})(34.0 \text{ s}) = \boxed{-221 \text{ m} = 221 \text{ m downward}}$

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**9.** $v_t = 10.0$ cm/s

$v_h = 20\ v_t = 2.00 \times 10^2$ cm/s

$\Delta t_{race} = \Delta t_t$

$\Delta t_h = \Delta t_t - 2.00$ min

$\Delta x_t = \Delta x_h + 20.0 \text{ cm} = \Delta x_{race}$

$\Delta x_t = v_t \Delta t_t$

$\Delta x_h = v_h \Delta t_h = v_h\,(\Delta t_t - 2.00 \text{ min})$

$\Delta x_t = \Delta x_{race} = \Delta x_h + 20.0 \text{ cm}$

$v_t\,\Delta t_t = v_h\,(\Delta t_t - 2.00 \text{ min}) + 20.0 \text{ cm}$

$\Delta t_t\,(v_t - v_h) = -v_h\,(2.00 \text{ min}) + 20.0 \text{ cm}$

$$\Delta t_t = \frac{20.0 \text{ cm} - v_h\,(2.00 \text{ min})}{v_t - v_h}$$

$$\Delta t_{race} = \Delta t_t = \frac{20.0 \text{ cm} - (2.00 \times 10^2 \text{ cm/s})(2.00 \text{ min})(60 \text{ s/min})}{10.0 \text{ cm/s} - 2.00 \times 10^2 \text{ cm/s}}$$

$$\Delta t_{race} = \frac{20.0 \text{ cm} - 2.40 \times 10^4 \text{ cm}}{-1.90 \times 10^2 \text{ cm/s}} = \frac{-2.40 \times 10^4 \text{ cm}}{-1.90 \times 10^2 \text{ cm/s}}$$

$\Delta t_{race} = 126$ s

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**10.** $\Delta x_{race} = \Delta x_t$

$vt = 10.0$ cm/s

$\Delta t_t = 126$ s

$\Delta x_{race} = \Delta x_t = v_t\Delta t_t = (10.0 \text{ cm/s})(126 \text{ s}) = \boxed{1.26 \times 10^3 \text{ cm} = 12.6 \text{ m}}$

## Additional Practice B

**1.** $\Delta t = 6.92$ s

$v_f = 17.34$ m/s

$v_i = 0$ m/s

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{17.34 \text{ m/s} - 0 \text{ m/s}}{6.92 \text{ s}} = \boxed{2.51 \text{ m/s}^2}$$

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**2.** $v_i = 0$ m/s

$v_f = 7.50 \times 10^2$ m/s

$\Delta t = 2.00$ min

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{7.50 \times 10^2 \text{ m/s} - 0 \text{ m/s}}{(2.00 \text{ min})\left(\frac{60 \text{ s}}{1 \text{ min}}\right)} = \boxed{6.25 \text{ m/s}^2}$$