NAME ______________________ DATE ____________ CLASS ____________

Motion in One Dimension

Problem C

DISPLACEMENT WITH CONSTANT ACCELERATION

PROBLEM

The arrow on a crossbow undergoes uniform acceleration over a distance of 38.1 cm. If the acceleration takes place over 8.93×10^{-3} s and the arrow is initially at rest, what is the arrow's final speed?

SOLUTION

Given: $\Delta x = 38.1$ cm

$\Delta t = 8.93 \times 10^{-3}$ s

$v_i = 0$ m/s

Unknown: $v_f = ?$

Use the equation for displacement with uniform acceleration.

$$\Delta x = \frac{1}{2}(v_i + v_f)\,\Delta t$$

Rearrange the equation to solve for v_f.

$$v_f = \frac{2\Delta x}{\Delta t} - v_i$$

$$v_f = \frac{(2)(38.1\text{ cm})\left(\dfrac{1\text{ m}}{100\text{ cm}}\right)}{8.93 \times 10^{-3}\text{ s}} - 0\text{ m/s}$$

$$v_f = 85.3\text{ m/s} - 0\text{ m/s}$$

$$v_f = \boxed{85.3\text{ m/s}}$$

ADDITIONAL PRACTICE

1. A device at Sandia Laboratories in Albuquerque, New Mexico, uses highly compressed air to accelerate small metal disks to supersonic speeds. Suppose the disk, which is initially at rest, undergoes a uniform acceleration for 0.910 s, at which point it reaches its top speed. If the disk travels 7.19 km in that time, what is its final speed?

2. Despite their size and awkward appearance, polar bears can run at respectable speeds for short distances. Suppose a polar bear running with an initial speed of 4.0 m/s accelerates uniformly for 18 s. What is the bear's maximum speed if the bear travels 135 m during the 18 s of acceleration? Give the answer in both meters per second and kilometers per hour?

3. A hockey puck slides 55.0 m along the length of the rink in just 1.25 s. The slight friction between the puck and the ice provides a uniform acceleration. If the puck's final speed is 43.2 m/s, what is its initial speed?

4. A child sleds down a snow-covered hill with a uniform acceleration. The slope of the hill is 38.5 m long. If the child starts at rest and reaches the bottom of the hill in 5.5 s, what is the child's final speed?

5. The longest stretch of straight railroad tracks lies across the desolate Nullarbor Plain, between the Australian cities of Adelaide and Perth. The tracks extend a distance of 478 km without a curve. Suppose a train with an initial speed of 72 km/h travels along the entire length of straight track with a uniform acceleration. The train reaches the end of the straight track in 5 h, 39 min. What is the train's final speed?

6. A golf ball at a miniature golf course travels 4.2 m along a carpeted green. When the ball reaches the hole 3.0 s later, its speed is 1.3 m/s. Assuming the ball undergoes constant uniform acceleration, what is the ball's initial speed?

7. A speedboat uniformly increases its velocity from 25 m/s to the west to 35 m/s to the west. How long does it take the boat to travel 250 m west while undergoing this acceleration?

8. Airplane racing, like horse and auto racing, uses a "track" of a specific length. Unlike the horse or auto tracks, the racing area for airplanes is bounded on the inside by tall columns, or pylons, around which the pilots must fly, and by altitude limitations that the pilots must monitor using their instruments. Different types of races use different arrangements of pylons to make the length of the race longer or shorter. In one particular race, a pilot begins the race at a speed of 755.0 km/h and accelerates at a constant uniform rate for 63.21 s. The pilot crosses the finish line with a speed of 777.0 km/h. From this data, calculate the length of the course.

9. A hovercraft, also known as an air-cushion vehicle, glides on a cushion of air, allowing it to travel with equal ease on land or water. Suppose a hovercraft undergoes constant uniform acceleration, which causes the hovercraft to move from rest to a speed of 30.8 m/s. How long does the hovercraft accelerate if it travels a distance of 493 m?

10. A spaceship travels 1220 km with a constant uniform acceleration. How much time is required for the acceleration if the spaceship increases its speed from 11.1 km/s to 11.7 km/s?

Givens | *Solutions*

3. $v_i = 0$ m/s

$v_f = 0.85$ m/s forward

$\Delta t = 3.7$ s

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{0.85 \text{ m/s} - 0 \text{ m/s}}{3.7 \text{ s}} = \boxed{0.23 \text{ m/s}^2 \text{ forward}}$$

4. $v_i = 13.7$ m/s forward $= +13.7$ m/s

$v_f = 11.5$ m/s backward $= -11.5$ m/s

$\Delta t = 0.021$ s

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{(-11.5 \text{ m/s}) - (13.7 \text{ m/s})}{0.021 \text{ s}} = \frac{-25.2 \text{ m/s}}{0.021 \text{ s}}$$

$$a_{avg} = \boxed{-1200 \text{ m/s}^2\text{, or } 1200 \text{ m/s}^2 \text{ backward}}$$

5. $v_i = +320$ km/h

$v_f = 0$ km/h

$\Delta t = 0.18$ s

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{(0 \text{ km/h} - 320 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)}{0.18 \text{ s}}$$

$$a_{avg} = \frac{-89 \text{ m/s}}{0.18 \text{ s}} = \boxed{-490 \text{ m/s}^2}$$

6. $a_{avg} = 16.5$ m/s^2

$v_i = 0$ km/h

$v_f = 386.0$ km/h

$$\Delta t = \frac{\Delta v}{a_{avg}} = \frac{v_f - v_i}{a_{avg}} = \frac{(386.0 \text{ km/h} - 0 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)}{16.5 \text{ m/s}^2}$$

$$\Delta t = \frac{107.2 \text{ m/s}}{16.5 \text{ m/s}^2} = \boxed{6.50 \text{ s}}$$

7. $v_i = -4.0$ m/s

$a_{avg} = -0.27$ m/s^2

$\Delta t = 17$ s

$$v_f = a_{avg}\Delta t + v_i$$

$$v_f = (-0.27 \text{ m/s}^2)(17 \text{ s}) + (-4.0 \text{ m/s}) = -4.6 \text{ m/s} - 4.0 \text{ m/s} = \boxed{-8.6 \text{ m/s}}$$

8. $v_i = 4.5$ m/s

$v_f = 10.8$ m/s

$a_{avg} = 0.85$ m/s^2

$$\Delta t = \frac{\Delta v}{a_{avg}} = \frac{v_f - v_i}{a_{avg}} = \frac{10.8 \text{ m/s} - 4.5 \text{ m/s}}{0.85 \text{ m/s}^2} = \frac{6.3 \text{ m/s}}{0.85 \text{ m/s}^2} = \boxed{7.4 \text{ s}}$$

9. $v_f = 296$ km/h

$v_i = 0$ km/h

$a_{avg} = 1.60$ m/s^2

$$\Delta t = \frac{\Delta v}{a_{avg}} = \frac{v_f - v_i}{a_{avg}} = \frac{(296 \text{ km/h} - 0 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)}{1.60 \text{ m/s}^2} = \frac{82.2 \text{ m/s}}{1.60 \text{ m/s}^2} = \boxed{51.4 \text{ s}}$$

10. $a_{avg} = -0.87$ m/s^2

$\Delta t = 3.85$

$$\Delta v = a_{avg}\Delta t = (-0.87 \text{ m/s}^2)(3.85 \text{ s}) = \boxed{-3.4 \text{ m/s}}$$

Additional Practice C

1. $\Delta t = 0.910$ s

$\Delta x = 7.19$ km

$v_i = 0$ km/s

$$v_f = \frac{2\Delta x}{\Delta t} - v_i = \frac{(2)(7.19 \text{ km})}{0.910 \text{ s}} - 0 \text{ km/s} = \boxed{15.8 \text{ km/s}}$$

Givens	*Solutions*
2. $v_i = 4.0$ m/s $\Delta t = 18$ s $\Delta x = 135$ m	$v_f = \frac{2\Delta x}{\Delta t} - v_i = \frac{(2)(135\text{ m})}{18\text{ s}} - 4.0\text{ m/s} = 15\text{ m/s} - 4.0\text{ m/s} = \boxed{11\text{ m/s}}$ $v_f = (11\text{ m/s})\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{1\text{ km}}{10^3\text{ m}}\right)$ $v_f = \boxed{4.0 \times 10^1\text{ km/h}}$
3. $\Delta x = 55.0$ m $\Delta t = 1.25$ s $v_f = 43.2$ m/s	$v_i = \frac{2\Delta x}{\Delta t} - v_f = \frac{(2)(55.0\text{ m})}{1.25\text{ s}} - 43.2\text{ m/s} = 88.0\text{ m/s} - 43.2\text{ m/s} = \boxed{44.8\text{ m/s}}$
4. $\Delta x = 38.5$ m $\Delta t = 5.5$ s $v_i = 0$ m/s	$v_f = \frac{2\Delta x}{\Delta t} - v_i = \frac{(2)(38.5\text{ m})}{5.5\text{ s}} - 0\text{ m/s} = \boxed{14\text{ m/s}}$
5. $\Delta x = 478$ km $\Delta v_i = 72$ km/h $\Delta t = 5$ h, 39 min	$v_f = \frac{2\Delta x}{\Delta t} - v_i = \frac{(2)(478\text{ km})}{5\text{ h} + 39\text{ min}\left(\frac{1\text{ h}}{60\text{ min}}\right)} - 72\text{ km/h} = \frac{(2)(478\text{ km})}{5\text{ h} + 0.65\text{ h}} - 72\text{ km/h}$ $v_f = \frac{956\text{ km}}{5.65\text{ h}} - 72\text{ km/h} = 169\text{ km/h} - 72\text{ km/h} = \boxed{97\text{ km/h}}$
6. $\Delta x = 4.2$ m $\Delta t = 3.0$ s $v_f = 1.3$ m/s	$v_i = \frac{2\Delta x}{\Delta t} - v_f = \frac{(2)(4.2\text{ m})}{3.0\text{ s}} - 1.3\text{ m/s} = 2.8\text{ m/s} - 1.3\text{ m/s} = \boxed{1.5\text{ m/s}}$
7. $v_i = 25$ m/s west $v_f = 35$ m/s west $\Delta x = 250$ m west	$\Delta t = \frac{2\Delta x}{v_i + v_f} = \frac{(2)(250\text{ m})}{25\text{ m/s} + 35\text{ m/s}} = \frac{5.0 \times 10^2\text{ m}}{6.0 \times 10^1\text{ m/s}} = \boxed{8.3\text{ s}}$
8. $v_i = 755.0$ km/h $v_f = 777.0$ km/h $\Delta t = 63.21$ s	$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t = \frac{1}{2}(755.0\text{ km/h} + 777.0\text{ km/h})(63.21\text{ s})\left(\frac{1\text{ h}}{3600\text{ s}}\right)$ $\Delta x = \frac{1}{2}(1532.0\text{ km/h})(1.756 \times 10^{-2}\text{ h}) = \boxed{13.45\text{ km}}$
9. $v_i = 0$ m/s $v_f = 30.8$ m/s $\Delta x = 493$ m	$\Delta t = \frac{2\Delta x}{v_i + v_f} = \frac{(2)(493\text{ m})}{0\text{ m/s} + 30.8\text{ m/s}} = \frac{986\text{ m}}{30.8\text{ m/s}} = \boxed{32.0\text{ s}}$
10. $\Delta x = 1220$ km $v_i = 11.1$ km/s $v_f = 11.7$ km/s	$\Delta t = \frac{2\Delta x}{v_i + v_f} = \frac{(2)(1220\text{ km})}{11.1\text{ km/s} + 11.7\text{ km/s}} = \frac{2440\text{ km}}{22.8\text{ km/s}} = \boxed{107\text{ s}}$