#### **Motion in One Dimension**

# Problem

#### **VELOCITY AND DISPLACEMENT WITH CONSTANT ACCELERATION**

#### **PROBLEM**

A barge moving with a speed of 1.00 m/s increases speed uniformly, so that in 30.0 s it has traveled 60.2 m. What is the magnitude of the barge's acceleration?

#### SOLUTION

Given:  $v_i = 1.00 \text{ m/s}$ 

 $\Delta t = 30.0 \text{ s}$ 

 $\Delta x = 60.2 \text{ m}$ 

**Unknown:** a = ?

Use the equation for displaement with constant uniform acceleration.

$$\Delta x = \nu_i \Delta t + \frac{1}{2} a \Delta t^2$$

Rearrange the equation to solve for a.

$$\frac{1}{2} a\Delta t^2 = \Delta x - \nu_i \Delta t$$

$$a = \frac{2(\Delta x - \nu_i \Delta t)}{\Delta t^2}$$

$$a = \frac{(2)[60.2 \text{ m} - (1.00 \text{ m/s})(30.0 \text{ s})]}{(30.0 \text{ s})^2}$$

$$a = \frac{(2)(60.2 \text{ m} - 30.0 \text{ m})}{9.00 \times 10^2 \text{ s}^2}$$

$$a = \frac{(2)(30.2 \text{ m})}{9.00 \times 10^2 \text{ s}^2}$$

$$a = 6.71 \times 10^{-2} \,\mathrm{m/s}^2$$

#### **ADDITIONAL PRACTICE**

- 1. The flight speed of a small bottle rocket can vary greatly, depending on how well its powder burns. Suppose a rocket is launched from rest so that it travels 12.4 m upward in 2.0 s. What is the rocket's net acceleration?
- **2.** The shark can accelerate to a speed of 32.0 km/h in a few seconds. Assume that it takes a shark 1.5 s to accelerate uniformly from 2.8 km/h to 32.0 km/h. What is the magnitude of the shark's acceleration?
- **3.** In order for the Wright brothers' 1903 flyer to reach launch speed, it had to be accelerated uniformly along a track that was 18.3 m long. A system of pulleys and falling weights provided the acceleration. If the flyer was initially at rest and it took 2.74 s for the flyer to travel the length of the track, what was the magnitude of its acceleration?

- **4.** A certain roller coaster increases the speed of its cars as it raises them to the top of the incline. Suppose the cars move at 2.3 m/s at the base of the incline and are moving at 46.7 m/s at the top of the incline. What is the magnitude of the net acceleration if it is uniform acceleration and takes place in 7.0 s?
- **5.** A ship with an initial speed of 6.23 m/s approaches a dock that is 255 m away. If the ship accelerates uniformly and comes to rest in 82 s, what is its acceleration?
- **6.** Although tigers are not the fastest of predators, they can still reach and briefly maintain a speed of 55 km/h. Assume that a tiger takes 4.1 s to reach this speed from an initial speed of 11 km/h. What is the magnitude of the tiger's acceleration, assuming it accelerates uniformly?
- **7.** Assume that a catcher in a professional baseball game catches a ball that has been pitched with an initial velocity of 42.0 m/s to the southeast. If the catcher uniformly brings the ball to rest in 0.0090 s through a distance of 0.020 m to the southeast, what is the ball's acceleration?
- **8.** A crate is carried by a conveyor belt to a loading dock. The belt speed uniformly increases slightly, so that for 28.0 s the crate accelerates by 0.035 m/s<sup>2</sup>. If the crate's initial speed is 0.76 m/s, what is its final speed?
- **9.** A plane starting at rest at the south end of a runway undergoes a uniform acceleration of  $1.60 \text{ m/s}^2$  to the north. At takeoff, the plane's velocity is 72.0 m/s to the north.
  - **a.** What is the time required for takeoff?
  - **b.** How far does the plane travel along the runway?
- **10.** A cross-country skier with an initial forward velocity of +4.42 m/s accelerates uniformly at -0.75 m/s<sup>2</sup>.
  - **a.** How long does it take the skier to come to a stop?
  - **b.** What is the skier's displacement in this time interval?

### **Additional Practice D**

#### Givens

#### Solutions

**1.** 
$$\Delta x = 12.4$$
 m upward  $\Delta t = 2.0$  s

Because 
$$v_i = 0$$
 m/s,  $a = \frac{2\Delta x}{\Delta t^2} = \frac{(2)(12.4 \text{ m})}{(2.0 \text{ s})^2} = \boxed{6.2 \text{ m/s}^2 \text{ upward}}$ 

 $v_i = 0 \text{ m/s}$ 

**2.** 
$$\Delta t = 1.5 \text{ s}$$
  
 $v_i = 2.8 \text{ km/h}$   
 $v_f = 32.0 \text{ km/h}$ 

$$a = \frac{v_f - v_i}{\Delta t} = \frac{(32.0 \text{ km/h} - 2.8 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) \left(\frac{10^3 \text{ m}}{\text{km}}\right)}{1.5 \text{ s}}$$

$$a = \frac{(29.2 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) \left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)}{1.5 \text{ s}} = \boxed{5.4 \text{ m/s}^2}$$

**3.** 
$$\Delta x = 18.3 \text{ m}$$

$$\Delta t = 2.74 \text{ s}$$

$$v_i = 0 \text{ m/s}$$

Because 
$$v_i = 0$$
 m/s,  $a = \frac{2\Delta x}{\Delta t^2} = \frac{(2)(18.3 \text{ m})}{(2.74 \text{ s})^2} = \boxed{4.88 \text{ m/s}^2}$ 

**4.** 
$$v_i = 2.3 \text{ m/s}$$

$$v_f = 46.7 \text{ m/s}$$

$$\Delta t = 7.0 \text{ s}$$

$$a = \frac{v_f - v_i}{\Delta t} = \frac{46.7 \text{ m/s} - 2.3 \text{ m/s}}{7.0 \text{ s}} = \frac{44.4 \text{ m/s}}{7.0 \text{ s}} = \boxed{6.3 \text{ m/s}^2}$$

$$\Delta t = 7.0 \text{ s}$$

**5.** 
$$v_i$$
 = 6.23 m/s

$$\Delta x = 255 \text{ m}$$

$$\Delta t = 82 \text{ s}$$

$$a = \frac{2(\Delta x - \nu_i \, \Delta t)}{\Delta t^2} = \frac{(2)[255 \text{ m} - (6.23 \text{ m/s})(82 \text{ s})]}{(82 \text{ s})^2}$$

$$a = \frac{(2)(255 \text{ m} - 510 \text{ m})}{6.7 \times 10^3 \text{ s}^2} = \frac{(2)(-255 \text{ m})}{6.7 \times 10^3 \text{ s}^2} = \boxed{-7.6 \times 10^{-2} \text{ m/s}^2}$$

**6.**  $v_i = 11 \text{ km/h}$ 

$$v_f = 55 \text{ km/h}$$

$$\Delta = 4.1 \text{ s}$$

$$a = \frac{v_f - v_i}{\Delta t} = \frac{(55 \text{ km/h} - 11 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) \left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)}{4.1 \text{ s}}$$

$$a = \frac{(44 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) \left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)}{4 \text{ l s}} = \boxed{3.0 \text{ m/s}^2}$$

**7.**  $v_i = 42.0 \text{ m/s southeast}$ 

$$\Delta t = 0.0090 \text{ s}$$

$$\Delta x = 0.020$$
 m/s southeast

$$a = \frac{2(\Delta x - \nu_i \Delta t)}{\Delta t^2} = \frac{(2)[0.020 \text{ m} - (42.0 \text{ m/s})(0.0090 \text{ s})]}{(0.0090 \text{ s})^2}$$

$$a = \frac{(2)(0.020 \text{ m/s} - 0.38 \text{ m})}{8.1 \times 10^{-5} \text{ s}^2} = \frac{(2)(-0.36 \text{ m})}{8.1 \times 10^{-5} \text{ s}^2}$$

$$a = \boxed{-8.9 \times 10^3 \text{ m/s}^2, \text{ or } 8.9 \times 10^3 \text{ m/s}^2 \text{ northwest}}$$

**8.**  $\Delta t = 28 \text{ s}$ 

$$a = 0.035 \text{ m/s}^2$$

$$v_i = 0.76 \text{ m/s}$$

 $v_f = a\Delta t + v_i = (0.035 \text{ m/s}^2)(28.0 \text{ s}) + 0.76 \text{ m/s} = 0.98 \text{ m/s} + 0.76 \text{ m/s} = 1.74 \text{ m/s}$ 

## Givens

## **9.** $v_i = 0 \text{ m/s}$ $v_f = 72.0 \text{ m/s north}$ $a = 1.60 \text{ m/s}^2 \text{ north}$ $\Delta t = 45.0 \text{ s}$

**a.** 
$$\Delta t = \frac{v_f - v_i}{a} = \frac{72.0 \text{ m/s} - 0 \text{ m/s}}{1.60 \text{ m/s}^2} = \boxed{45.0 \text{ s}}$$

**b.** 
$$\Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2 = (0 \text{ m/s})(45.0 \text{ s}) + \frac{1}{2} (1.60 \text{ m/s}^2)(45.0 \text{ s})^2 = 0 \text{ m} + 1620 \text{ m}$$
  
 $\Delta x = \boxed{1.62 \text{ km}}$ 

**10.** 
$$v_i$$
 = +4.42 m/s  
 $v_f$  = 0 m/s  
 $a$  = -0.75 m/s<sup>2</sup>  
 $\Delta t$  = 5.9 s

**a.** 
$$\Delta t = \frac{v_f - v_i}{a} = \frac{0 \text{ m/s} - 4.42 \text{ m/s}}{-0.75 \text{ m/s}^2} = \frac{-4.42 \text{ m/s}}{-0.75 \text{ m/s}^2} = \boxed{5.9 \text{ s}}$$

**b.** 
$$\Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2 = (4.42 \text{ m/s})(5.9 \text{ s}) + \frac{1}{2} (-0.75 \text{ m/s}^2)(5.9 \text{ s})^2$$
  
 $\Delta x = 26 \text{ m} - 13 \text{ m} = \boxed{13 \text{ m}}$ 

## **Additional Practice E**

**1.** 
$$v_i = 1.8 \text{ km/h}$$
  
 $v_f = 24.0 \text{ km/h}$   
 $\Delta x = 4.0 \times 10^2 \text{ m}$ 

$$a = \frac{v_f^2 - v_i^2}{2\Delta x} = \frac{\left[ (24.0 \text{ km/h})^2 - (1.8 \text{ km/h})^2 \right] \left( \frac{1 \text{ h}}{3600 \text{ s}} \right)^2 \left( \frac{10^3 \text{ m}}{1 \text{ km}} \right)^2}{(2)(4.0 \times 10^2 \text{ m})}$$

$$a = \frac{(576 \text{ km}^2/\text{h}^2 - 3.2 \text{ km}^2/\text{h}^2) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right)^2 \left( \frac{10^3 \text{ m}}{1 \text{ km}} \right)^2}{8.0 \times 10^2 \text{ m}}$$

$$a = \frac{(573 \text{ km}^2/\text{h}^2) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right)^2 \left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)^2}{8.0 \times 10^2 \text{ m}} = \boxed{5.5 \times 10^{-2} \text{ m/s}^2}$$

**2.** 
$$v_f = 0 \text{ m/s}$$
  
 $v_f = 8.57 \text{ m/s}$   
 $\Delta x = 19.53 \text{ m}$ 

$$a = \frac{v_f^2 - v_i^2}{2\Delta x} = \frac{(8.57 \text{ m/s})^2 - (0 \text{ m/s})^2}{(2)(19.53 \text{ m})} = \frac{73.4 \text{ m}^2/\text{s}^2}{39.06 \text{ m}} = \boxed{1.88 \text{ m/s}^2}$$

**3.** 
$$v_i = 7.0 \text{ km/h}$$
  
 $v_f = 34.5 \text{ km/h}$   
 $\Delta x = 95 \text{ m}$ 

$$a = \frac{v_f^2 - v_i^2}{2\Delta x} = \frac{\left[ (34.5 \text{ km/h})^2 - (7.0 \text{ km/h})^2 \right] \left( \frac{1 \text{ h}}{3600 \text{ s}} \right)^2 \left( \frac{10^3 \text{ m}}{1 \text{ km}} \right)^2}{(2)(95 \text{ m})}$$

$$a = \frac{(1190 \text{ km}^2/\text{h}^2 - 49 \text{ km}^2/\text{h}^2) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right)^2 \left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)^2}{190 \text{ m}}$$

$$a = \frac{(1140 \text{ km}^2/\text{h}^2) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right)^2 \left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)^2}{190 \text{ m}} = \boxed{0.46 \text{ m/s}^2}$$

**4.** 
$$\Delta x = 2.00 \times 10^2 \text{ m}$$

$$a = \frac{v_f^2 - v_i^2}{2\Delta x} = \frac{(10.22 \text{ m/s})^2 - (9.78 \text{ m/s})^2}{(2)(2.00 \times 10^2 \text{ m})} = \frac{104.4 \text{ m}^2/\text{s}^2 - 95.6 \text{ m}^2/\text{s}^2}{4.00 \times 10^2 \text{ m}}$$

$$v_i = 9.78 \text{ m/s}$$
  
 $v_f = 10.22 \text{ m/s}$ 

$$a = \frac{8.8 \text{ m}^2/\text{s}^2}{4.00 \times 10^2 \text{ m}} = \boxed{2.2 \times 10^{-2} \text{ m/s}^2}$$