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Motion in One Dimension

Problem D

VELOCITY AND DISPLACEMENT WITH CONSTANT ACCELERATION

PROBLEM

Some cockroaches can run as fast as 1.5 m/s. Suppose that two cockroaches are separated by a distance of 60.0 cm and that they begin to run toward each other at the same moment. Both insects have constant acceleration until they meet. The first cockroach has an acceleration of 0.20 m/s^2 in one direction, and the second one has an acceleration of 0.12 m/s^2 in the opposite direction. How much time passes before the two insects bump into each other?

SOLUTION

1. DEFINE

Given: $a_1 = 0.20$ m/s^2 (first cockroach's acceleration)
$v_{i,1} = 0$ m/s (first cockroach's initial speed)
$a_2 = 0.12$ m/s^2 (second cockroach's acceleration)
$v_{i,2} = 0$ m/s (second cockroach's initial speed)
$d = 60.0$ cm $= 0.60$ m (initial distance between the insects)

Unknown: $\Delta x_1 = ?$ $\Delta x_2 = ?$ $\Delta t = ?$

2. PLAN

Choose an equation(s) or situation: Use the equation for displacement with constant acceleration for each cockroach.

$$\Delta x_1 = v_{i,1}\Delta t + \tfrac{1}{2}a_1\Delta t^2$$

$$\Delta x_2 = v_{i,2}\Delta t + \tfrac{1}{2}a_2\Delta t^2$$

The distance the second cockroach travels can be expressed as the difference between the total distance that initially separates the two insects and the distance that the first insect travels.

$$\Delta x_2 = d - \Delta x_1$$

Rearrange the equation(s) to isolate the unknown(s): Substitute the expression for the first cockroach's displacement into the equation for the second cockroach's displacement using the equation relating the two displacements to the initial distance between the insects.

$$\Delta x_2 = d - \Delta x_1 = d - \left(v_{i,1}\Delta t + \tfrac{1}{2}a_1\Delta t^2\right)$$

$$= v_{i,2}\Delta t + \tfrac{1}{2}a_2\Delta t^2$$

The equation can be rewritten to express Δt in terms of the known quantities. To simplify the calculation, the terms involving the initial speeds, which are both zero, can be removed from the equations.

$$d - \tfrac{1}{2}a_1\Delta t^2 = \tfrac{1}{2}a_2\Delta t^2$$

$$\Delta t = \sqrt{\frac{2d}{a_1 + a_2}}$$

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3. CALCULATE **Substitute the values into the equation(s) and solve:**

$$\Delta t = \sqrt{\frac{(2)(0.60\text{ m})}{0.20\frac{\text{m}}{\text{s}^2} + 0.12\frac{\text{m}}{\text{s}^2}}} = \sqrt{\frac{1.2\text{ m}}{0.32\text{ m/s}^2}} = \boxed{1.9\text{ s}}$$

4. EVALUATE The final speeds for the first and second cockroaches are 0.38 m/s and 0.23 m/s, respectively. Both of these values are well below the maximum speed for cockroaches in general.

ADDITIONAL PRACTICE

1. In 1986, the first flight around the globe without a single refueling was completed. The aircraft's average speed was 186 km/h. If the airplane landed at this speed and accelerated at -1.5 m/s^2, how long did it take for the airplane to stop?

2. In 1976, Gerald Hoagland drove a car over 8.0×10^2 km in reverse. Fortunately for Hoagland and motorists in general, the event took place on a special track. During this drive, Hoagland's average velocity was about –15.0 m/s. Suppose Hoagland decides during his drive to go forward. He applies the brakes, stops, and then accelerates until he moves forward at same speed he had when he was moving backward. How long would the entire reversal process take if the average acceleration during this process is $+2.5\text{ m/s}^2$?

3. The first permanent public railway was built by George Stephenson and opened in Cleveland, Ohio, in 1825. The average speed of the trains was 24.0 km/h. Suppose a train moving at this speed accelerates -0.20 m/s^2 until it reaches a speed of 8.0 km/h. How long does it take the train to undergo this change in speed?

4. The winding cages in mine shafts are used to move workers in and out of the mines. These cages move much faster than any commercial elevators. In one South African mine, speeds of up to 65.0 km/h are attained. The mine has a depth of 2072 m. Suppose two cages start their downward journey at the same moment. The first cage quickly attains the maximum speed (an unrealistic situation), then proceeds to descend uniformly at that speed all the way to the bottom. The second cage starts at rest and then increases its speed with a constant acceleration of magnitude $4.00 \times 10^{-2}\text{ m/s}^2$. How long will the trip take for each cage? Which cage will reach the bottom of the mine shaft first?

5. In a 1986 bicycle race, Fred Markham rode his bicycle a distance of 2.00×10^2 m with an average speed of 105.4 km/h. Markham and the bicycle started the race with a certain initial speed.
   a. Find the time it took Markham to cover 2.00×10^2 m.
   b. Suppose a car moves from rest under constant acceleration. What is the magnitude of the car's acceleration if the car is to finish the race at exactly the same time Markham finishes the race?

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6. Some tropical butterflies can reach speeds of up to 11 m/s. Suppose a butterfly flies at a speed of 6.0 m/s while another flying insect some distance ahead flies in the same direction with a constant speed. The butterfly then increases its speed at a constant rate of 1.4 m/s^2 and catches up to the other insect 3.0 s later. How far does the butterfly travel during the race?

7. Mary Rife, of Texas, set a women's world speed record for sailing. In 1977, her vessel, *Proud Mary,* reached a speed of 3.17×10^2 km/h. Suppose it takes 8.0 s for the boat to decelerate from 3.17×10^2 km/h to 2.00×10^2 km/h. What is the boat's acceleration? What is the displacement of the *Proud Mary* as it slows down?

8. In 1994, a human-powered submarine was designed in Boca Raton, Florida. It achieved a maximum speed of 3.06 m/s. Suppose this submarine starts from rest and accelerates at 0.800 m/s^2 until it reaches maximum speed. The submarine then travels at constant speed for another 5.00 s. Calculate the total distance traveled by the submarine.

9. The highest speed achieved by a standard nonracing sports car is 3.50×10^2 km/h. Assuming that the car accelerates at 4.00 m/s^2, how long would this car take to reach its maximum speed if it is initially at rest? What distance would the car travel during this time?

10. Stretching 9345 km from Moscow to Vladivostok, the Trans-Siberian railway is the longest single railroad in the world. Suppose the train is approaching the Moscow station at a velocity of +24.7 m/s when it begins a constant acceleration of –0.850 m/s^2. This acceleration continues for 28 s. What will be the train's final velocity when it reaches the station?

11. The world's fastest warship belongs to the United States Navy. This vessel, which floats on a cushion of air, can move as fast as 1.7×10^2 km/h. Suppose that during a training exercise the ship accelerates +2.67 m/s^2, so that after 15.0 s its displacement is $+6.00 \times 10^2$ m. Calculate the ship's initial velocity just before the acceleration. Assume that the ship moves in a straight line.

12. The first supersonic flight was performed by then Capt. Charles Yeager in 1947. He flew at a speed of 3.00×10^2 m/s at an altitude of more than 12 km, where the speed of sound in air is slightly less than 3.00×10^2 m/s. Suppose Capt. Yeager accelerated 7.20 m/s^2 in 25.0 s to reach a final speed of 3.00×10^2 m/s. What was his initial speed?

13. Peter Rosendahl rode his unicycle a distance of 1.00×10^2 m in 12.11 s. If Rosendahl started at rest, what was the magnitude of his acceleration?

14. Suppose that Peter Rosendahl began riding the unicycle with a speed of 3.00 m/s and traveled a distance of 1.00×10^2 m in 12.11s. What would the magnitude of Rosendahl's acceleration be in this case?

15. In 1991, four English teenagers built an electric car that could attain a speed 30.0 m/s. Suppose it takes 8.0 s for this car to accelerate from 18.0 m/s to 30.0 m/s. What is the magnitude of the car's acceleration?

Givens *Solutions*

4. $v_i = v_{avg} = 518$ km/h

$v_f = (0.600)\ v_{avg}$

$\Delta t = 2.00$ min

$$v_{avg} = \left(518\frac{\text{km}}{\text{h}}\right)\left(\frac{1\text{ h}}{60\text{ min}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right) = 8.63 \times 10^3\text{ m/min}$$

$$\Delta x = \tfrac{1}{2}(v_i + v_f)\Delta t = \tfrac{1}{2}[v_{avg} + (0.600)\ v_{avg}]\Delta t = \tfrac{1}{2}(1.600)(8.63 \times 10^3\text{ m/min})(2.00\text{ min})$$

$$\Delta x = 13.8 \times 10^3\text{ m} = \boxed{13.8\text{ km}}$$

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5. $\Delta t = 30.0$ s

$v_i = 30.0$ km/h

$v_f = 42.0$ km/h

$$\Delta x = \tfrac{1}{2}(v_i + v_f)\Delta t = \tfrac{1}{2}(30.0\text{ km/h} + 42.0\text{ km/h})\left(\frac{1\text{h}}{3600\text{ s}}\right)(30.0\text{ s})$$

$$\Delta x = \tfrac{1}{2}\left(72.0\frac{\text{km}}{\text{h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)(30.0\text{ s})$$

$$\Delta x = 3.00 \times 10^{-1}\text{ km} = \boxed{3.00 \times 10^2\text{ m}}$$

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6. $v_f = 96$ km/h

$v_i = 0$ km/h

$\Delta t = 3.07$ s

$$\Delta x = \tfrac{1}{2}(v_i + v_f)\Delta t = \tfrac{1}{2}(0\text{ km/h} + 96\text{ km/h})\left(\frac{1\text{h}}{3600\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)(3.07\text{ s})$$

$$\Delta x = \tfrac{1}{2}\left(96 \times 10^3\frac{\text{m}}{\text{h}}\right)(8.53 + \times 10^{-4}\text{ h}) = \boxed{41\text{ m}}$$

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7. $\Delta x = 290.0$ m

$\Delta t = 10.0$ s

$v_f = 0$ km/h $= 0$ m/s

$$v_i = \frac{2\Delta x}{\Delta t} - v_f = \frac{(2)(290.0\text{ m})}{10.0\text{ s}} - 0\text{ m/s} = \boxed{58.0\text{ m/s} = 209\text{ km/h}}$$

(Speed was in excess of 209 km/h.)

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8. $\Delta x = 5.7 \times 10^3$ km

$\Delta t = 86$ h

$v_f = v_i + (0.10)\ v_i$

$$v_f + v_i = \frac{2\Delta x}{\Delta t}$$

$$v_i\ (1.00 + 0.10) + v_i = \frac{2\Delta x}{\Delta t}$$

$$v_i = \frac{(2)(5.7 \times 10^3\text{ km})}{(2.10)(86\text{ h})} = \boxed{63\text{ km/h}}$$

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9. $v_i = 2.60$ m/s

$v_f = 2.20$ m/s

$\Delta t = 9.00$ min

$$\Delta x = \tfrac{1}{2}(v_i + v_f)\Delta t = \tfrac{1}{2}(2.60\text{ m/s} + 2.20\text{ m/s})(9.00\text{ min})\left(\frac{60\text{ s}}{\text{min}}\right) = \tfrac{1}{2}(4.80\text{ m/s})(5.40 \times 10^2\text{ s})$$

$$\Delta x = 1.30 \times 10^3\text{ m} = \boxed{1.30\text{ km}}$$

Additional Practice D

1. $v_i = 186$ km/h

$v_f = 0$ km/h $= 0$ m/s

$a = -1.5$ m/s^2

$$\Delta t = \frac{v_f - v_i}{a} = \frac{0\text{ m/s} - (186\text{ km/h})\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)}{-1.5\text{ m/s}^2} = \frac{-51.7\text{ m/s}}{-1.5\text{ m/s}^2} = \boxed{34\text{ s}}$$

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2. $v_i = -15.0$ m/s

$v_f = 0$ m/s

$a = +2.5$ m/s^2

For stopping:

$$\Delta t_1 = \frac{v_f - v_i}{a} = \frac{0\text{ m/s} - (-15.0\text{ m/s})}{2.5\text{ m/s}^2} = \frac{15.0\text{ m/s}}{2.5\text{ m/s}^2} = 6.0\text{ s}$$

For moving forward:

$v_i = 0$ m/s

$v_f = +15.0$ m/s

$a = +2.5$ m/s

$$\Delta t_2 = \frac{v_f - v_i}{a} = \frac{15.0\text{ m/s} - 0.0\text{ m/s}}{2.5\text{ m/s}^2} = \frac{15.0\text{ m/s}}{2.5\text{ m/s}^2} = 6.0\text{ s}$$

$$\Delta t_{tot} = \Delta t_1 + \Delta t_2 = 6.0\text{ s} + 6.0\text{ s} = \boxed{12.0\text{ s}}$$

Givens *Solutions*

3. $v_i = 24.0 \text{ km/h}$

$v_f = 8.0 \text{ km/h}$

$a = -0.20 \text{ m/s}^2$

$$\Delta t = \frac{v_f - v_i}{a}$$

$$\Delta t = \frac{(8.0 \text{ km/h} - 24.0 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)}{-0.20 \text{ m/s}^2}$$

$$\Delta t = \frac{\left(-16.0 \frac{\text{km}}{\text{h}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)}{-0.20 \text{ m/s}^2} = \boxed{22 \text{ s}}$$

4. $v_1 = 65.0 \text{ km/h}$

$v_{i,2} = 0 \text{ km/h}$

$a_2 = 4.00 \times 10^{-2} \text{ m/s}^2$

$\Delta x = 2072 \text{ m}$

For cage 1:

$$\Delta x = v_1 \Delta t_1$$

$$\Delta t_1 = \frac{\Delta x}{v_1} = \frac{2072 \text{ m}}{(65.0 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)} = \boxed{115 \text{ s}}$$

For cage 2:

$$\Delta x = v_{i,2}\Delta t_2 + \tfrac{1}{2} a_2 \Delta t_2^2$$

Because $v_{i,2} = 0 \text{ km/h}$,

$$\Delta t_2 = \sqrt{\frac{2\Delta x}{a_2}} = \sqrt{\frac{(2)(2072 \text{ m})}{4.00 \times 10^{-2} \text{ m/s}^2}} = \boxed{322 \text{ s}}$$

Cage 1 reaches the bottom of the shaft in nearly a third of the time required for cage 2.

5. $\Delta x = 2.00 \times 10^2 \text{ m}$

$v = 105.4 \text{ km/h}$

a. $$\Delta t = \frac{\Delta x}{v} = \frac{2.00 \times 10^2 \text{ m}}{\left(105.4 \frac{\text{km}}{\text{h}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)} = \boxed{6.83 \text{ s}}$$

$v_{i,car} = 0 \text{ m/s}$

b. $$\Delta x = v_{i,car}\Delta t + \tfrac{1}{2} a_{car} \Delta t^2$$

$$a_{car} = \frac{2\Delta x}{\Delta t^2} = \frac{(2)(2.00 \times 10^2 \text{ m})}{(6.83 \text{ s})^2} = \boxed{8.57 \text{ m/s}^2}$$

6. $v_i = 6.0 \text{ m/s}$

$a = 1.4 \text{ m/s}^2$

$\Delta t = 3.0 \text{ s}$

$$\Delta x = v_i \Delta t + \tfrac{1}{2} a \Delta t^2 = (6.0 \text{ m/s})(3.0 \text{ s}) + \tfrac{1}{2}(1.4 \text{ m/s}^2)(3.0 \text{ s})^2 = 18 \text{ m} + 6.3 \text{ m} = \boxed{24 \text{ m}}$$

7. $v_i = 3.17 \times 10^2 \text{ km/h}$

$v_f = 2.00 \times 10^2 \text{ km/h}$

$\Delta t = 8.0 \text{ s}$

$$a = \frac{v_f - v_i}{\Delta t} = \frac{(2.00 \times 10^2 \text{ km/h} - 3.17 \times 10^2 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)}{8.0 \text{ s}}$$

$$a = \frac{(-117 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)}{8.0 \text{ s}} = \boxed{-4.1 \text{ m/s}^2}$$

$$\Delta x = v_i \Delta t + \tfrac{1}{2} a \Delta t^2 = (3.17 \times 10^2 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)(8.0 \text{ s}) + \tfrac{1}{2}(-4.1 \text{ m/s}^2)(8.0 \text{ s})^2$$

$$\Delta x = (7.0 \times 10^2 \text{ m}) + (-130 \text{ m}) = \boxed{+570 \text{ m}}$$

Givens *Solutions*

8. $v_i = 0$ m/s
$v_f = 3.06$ m/s
$a = 0.800$ m/s^2
$\Delta t_2 = 5.00$ s

$$\Delta t_1 = \frac{v_f - v_i}{a} = \frac{3.06 \text{ m/s} - 0 \text{ m/s}}{0.800 \text{ m/s}^2} = 3.82$$

$$\Delta x_1 = v_i \Delta t_1 + \tfrac{1}{2} a \Delta t_1{}^2 = (0 \text{ m/s})(3.82 \text{ s}) + \tfrac{1}{2}(0.800 \text{ m/s}^2)(3.82 \text{ s})^2 = 5.84 \text{ m}$$

$$\Delta x_2 = v_f \Delta t_2 = (3.06 \text{ m/s})(5.00 \text{ s}) = 15.3 \text{ m}$$

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 5.84 \text{ m} + 15.3 \text{ m} = \boxed{21.1 \text{ m}}$$

9. $v_f = 3.50 \times 10^2$ km/h
$v_i = 0$ km/h $= 0$ m/s
$a = 4.00$ m/s^2

$$\Delta t = \frac{(v_f - v_i)}{a} = \frac{(3.50 \times 10^2 \text{ km/h} - 0 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)}{(4.00 \text{ m/s}^2)} = \boxed{24.3 \text{ s}}$$

$$\Delta x = v_i \Delta t + \tfrac{1}{2} a \Delta t^2 = (0 \text{ m/s})(24.3 \text{ s}) + \tfrac{1}{2}(4.00 \text{ m/s}^2)(24.3 \text{ s})^2$$

$$\Delta x = 1.18 \times 10^3 \text{ m} = \boxed{1.18 \text{ km}}$$

10. $v_i = 24.0$ m/s
$a = -0.850$ m/s^2
$\Delta t = 28.0$ s

$$v_f = v_i + a\Delta t = 24.0 \text{ m/s} + (-0.850 \text{ m/s}^2)(28.0 \text{ s}) = 24.0 \text{ m/s} - 23.8 \text{ m/s} = \boxed{+0.2 \text{ m/s}}$$

11. $a = +2.67$ m/s^2
$\Delta t = 15.0$ s
$\Delta x = +6.00 \times 10^2$ m

$$v_i \Delta t = \Delta x - \tfrac{1}{2} a \Delta t^2$$

$$v_i = \frac{\Delta x}{\Delta t} - \tfrac{1}{2} a \Delta t = \frac{6.00 \times 10^2 \text{ m}}{15.0 \text{ s}} - \tfrac{1}{2}(2.67 \text{ m/s}^2)(15.0 \text{ s}) = 40.0 \text{ m/s} - 20.0 \text{ m/s} = \boxed{+20.0 \text{ m/s}}$$

12. $a = 7.20$ m/s^2
$\Delta t = 25.0$ s
$v_f = 3.00 \times 10^2$ ms

$$v_i = v_f - a\Delta t$$

$$v_i = (3.00 \times 10^2 \text{ m/s}) - (7.20 \text{ m/s}^2)(25.0 \text{ s}) = (3.00 \times 10^2 \text{ m/s}) - (1.80 \times 10^2 \text{ m/s})$$

$$v_i = \boxed{1.20 \times 10^2 \text{ m/s}}$$

13. $v_i = 0$ m/s
$\Delta x = 1.00 \times 10^2$ m
$\Delta t = 12.11$ s

$$\Delta x = v_i \Delta t + \tfrac{1}{2} a \Delta t^2$$

Because $v_i = 0$ m/s,

$$a = \frac{2\Delta x}{\Delta t^2} = \frac{(2)(1.00 \times 10^2 \text{ m})}{(12.11 \text{ s})^2} = \boxed{1.36 \text{ m/s}^2}$$

14. $v_i = 3.00$ m/s
$\Delta x = 1.00 \times 10^2$ m
$\Delta t = 12.11$ s

$$a = \frac{2(\Delta x - v_i \Delta t)}{\Delta t^2} = \frac{(2)[1.00 \times 10^2 \text{ m} - (3.00 \text{ m/s})(12.11 \text{ s})]}{(12.11 \text{ s})^2}$$

$$a = \frac{(2)(1.00 \times 10^2 \text{ m} - 36.3 \text{ m})}{(12.11 \text{ s})^2}$$

$$a = \frac{(2)(64 \text{ m})}{(12.11 \text{ s})^2} = \boxed{0.87 \text{ m/s}^2}$$

15. $v_f = 30.0$ m/s
$v_i = 18.0$ m/s
$\Delta t = 8.0$ s

$$a = \frac{v_f - v_i}{\Delta t} = \frac{30.0 \text{ m/s} - 18.0 \text{ m/s}}{8.0 \text{ s}} = \frac{12.0 \text{ m/s}}{8.0 \text{ s}} = \boxed{1.5 \text{ m/s}^2}$$

II