Chapter 1 Test Study Guide: Functions and Their Graphs

Section 1: Rectangular Coordinates

> ➢ Know the coordinate plane
> ➢ Plot points
> ➢ Distance Formula
> ➢ Midpoint Formula

Examples:

1. Draw the coordinate plane. Label the x- and y-axes, the origin, and the quadrants.

2. Plot the points (-2, 6) and (4, -3)

   a) Find the distance between the points

   b) Find the midpoint of the line segment joining the points

3. Determine the quadrant(s) in which (x, y) is located so that the conditions are satisfied.

   a) $x > 0$, $y < 0$

   b) $x < 0$, $xy > 0$

Additional problems from book: page 117 #1 – 4, page 9 #1 – 6, 11 – 20, 27 – 40

Section 2: Graphs of Equations

> ➢ Find x- and y-intercepts of the graph of the equation

Examples:

4. $f(x) = \frac{1}{3}x + 2$

5. $y = |3x - 5| - 4$

6. $g(x) = \sqrt{2x + 1} + 6$

7. $h(x) = \sqrt{2x - 1} - 6$

8. $y = (x - 2)^2 - 3$

9. $y = x^3 - 3x^2 - 25x + 75$

Additional problems from book: page 117 #25 – 28, page 22 #9 – 20

Section 3: Linear Equations in Two Variables

  ➢ Find slope between two points
  ➢ Use slope to graph lines
  ➢ Graph lines
  ➢ Write linear equations in two variables
  ➢ Use slope to identify parallel and perpendicular lines

Examples:

10. Find the slope of the line passing through the points (3, -4) and (-7, 1)

11. Graph the lines

  a) $2x - 5y = 15$

  b) $y = -2$

  c) $2x - 5 = 11$

12. Write the equation of each line with the given characteristics

  a) contains (3, -2) and has slope of $\frac{3}{2}$

  b) has x-intercept of 5 and slope of $-\frac{1}{2}$

  c) passes through (3, -2) and (3, 5)

  d) contains $f(2) = 5$ and $f(-2) = -1$

  e) passes through (1, -5) and is parallel to line with equation $3x - 5y = 10$

  f) passes through (-3, -7) and is perpendicular to the line with equation $2x - 7y = 7$

 g) contains (5, -4) and is perpendicular to the line with equation $y = 6$

  Additional problems from book: page 118 #47 – 64, page 123 #7 – 9

Section 4: Functions

  ➢ Determine whether relations between two variables are functions – graphically and by equation
  ➢ Use function notation and evaluate functions, including piecewise functions
  ➢ State the domain of the graph of an equation – graphically and by equation
  ➢ Evaluate difference quotients

Examples:

13. Use the Vertical Line Test to determine whether y is a function of x.
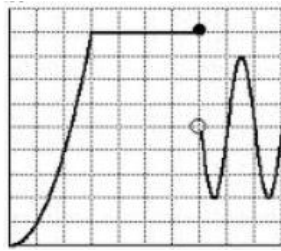
a)
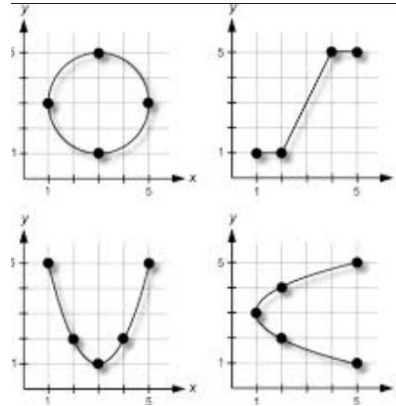
b - e)

14. Determine whether the equation represents y as a function of x.

   a) $2x - y = 5$

   b) $y = \sqrt{x - 3}$

   c) $y = \frac{1}{x+3} + 2$

   d) $x + y^2 = 16$

   e) $x^2 - y = 4$

15. State the domain of each function.

   a) $5x - 2y = 12$

   b) $y = 2(x - 3)^2 - 1$

   c) $y = \frac{2}{x^2 - 8x + 12} - 3$

   d) $y = \sqrt{x^2 - 25}$

   e) $y = \sqrt[3]{x - 6}$

16. Evaluate the function at each specified value.

   $f(x) = x^2 - x + 2$

   a) $f(3)$ b) $f(-2)$ c) $f\left(\frac{1}{3}\right)$ d) $f(x + 3)$

17. Evaluate the piecewise function at each specified value.

   $f(x) = -5x - 2, x \leq -2 \; ; \; x^2 - 2, x > -2$

   a) $f(3)$ b) $f(-2)$ c) $f\left(\frac{1}{3}\right)$ d) $f(-3)$

18. Find each difference quotient and simplify your answer.

   a) $given \; f(x) = x^2 - 2x + 9, \; find \; \frac{f(3+h)-f(3)}{h}, h \neq 0$

   b) $given \; f(x) = x^2 - 6x + 5, find \; \frac{f(x+h)-f(x)}{h}, h \neq 0$

Additional problems from book:
page 119 #67 – 76, 79, 81 – 84(state the domain and range of each also), 103 – 104
page 123 #10, 11
page 65 #115 – 116

Section 5: Analyzing Graphs of Functions

> ➢ Find the zeros of functions
> ➢ Determine the average rate of change of a function
> ➢ Identify even or odd functions
> ➢ State where a function is increasing, decreasing, or constant

Examples:

19. Find the zeros of the functions.

     a) $f(x) = 3x^2 - 7x + 2$

     b) $g(x) = \sqrt{5x - 2} - 5$

     c) $h(x) = \frac{x-5}{2x+1}$

     d) $k(x) = |x - 1| - 6$

20. Find the average rate of change of the function from x_1 to x_2.

     a) $f(x) = 5x - 6, \quad x_1 = 3, \ x_2 = 5$

     b) $f(x) = x^3 + 12x - 2, \quad x_1 = 0, \ x_2 = 2$

21. If the point (3, -2) is on an odd function, what other point is on the function?

22. If the point (3, -2) is on an even function, what other point is on the function?

23. Given the piecewise-defined function in #17. State the intervals in which the function is increasing, decreasing, or constant.

Additional problems from book: page 119 #85 – 88, 95, 96 and page 123 #12 – 14

Section 6: A Library of Parent Functions

> ➢ Identify and graph the parent functions which include: absolute value, squaring, square root, cubic, reciprocal, and greatest integer functions

Section 7: Transformations of Functions

> ➤ Use vertical and horizontal shifts to sketch graphs
> ➤ Use reflections to sketch graphs
> ➤ Use non-rigid transformations to sketch graphs

Examples:

24.   If A(5, -4) is on the graph of f, state the coordinates of A' on the graph of

g(x) = -3f(2x − 6) + 1.

25.   Describe all shifts, reflections and transformations then graph $h(x) = -2\sqrt{\frac{1}{3}x - 2} + 1$

Additional problems from book: page 120 #105 – 129 odds,

Section 8: Combinations of Functions: Composite Functions

> ➤ Add, subtract, multiply and divide functions
> ➤ Find the composition of functions
> ➤ State restrictions on the domain of composition of functions

Examples:

26.   Given $f(x) = \frac{1}{2}x + 3\ and\ g(x) = 5x - 1$. Find the following.
   a) $(f - g)(x)$
   b) $\left(\frac{f}{g}\right)(x)$
   c) $(f \cdot g)(2)$
   d) $(f \circ g)(x)$
   e) $(g \circ f)(-4)$

Additional problems from book: page 120 #131, 133, 135

Section 9: Inverse Functions

> ➤ Find the inverse of a function
> ➤ Verify inverse functions graphically (mirror images over y = x) and using compositions $(f(g(x)) = x = g(f(x)))$
> ➤ Use graphs of functions to determine whether functions have inverse functions
> ➤ Horizontal line test for one-to-one function

Examples:

27.   Find the inverse of the function $f(x) = 5x - 3$
28.   Find the inverse of the function $g(x) = \sqrt{-x - 5} - 3$

Additional problems from book: page 121 #141, 147, 149, 151