**Content Area:** Mathematics **Grade Level:** 11

**Content Standard:** M11.C Geometry

**PASA Anchor:**M11.CA.1.3 Use basic properties of two- and three-dimensional figures (sort).

**PASA linked to PSSA Anchor(s):** M11.C.1.3 Use propreties of congruence, correspondence and similarity in problem-solving settings involving two- and three-dimensional figures (Reference: 2.9.11.B).

**Grade Level PSSA Anchor/Eligible Content:** M11C.1.3.1 Identify and/or use properties of congruent and similar polygons or solids

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| --- | --- |
| **Webb’s Depth of Knowledge (Cognitive Demand)** | |
| **1 – Recall**  **2 – Application of Skill/Concept** | **3 – Strategic Thinking**  **4 – Extended Thinking** |

**Big Idea:** Similarity relationships between objects are a form of proportional relationships. Congruence describes a special similarity relationship between objects and is a form of equivalence.

Spatial reasoning and visualization are ways to orient thinking about the physical world.

**Essential Questions:** What are the key properties needed to determine a unique triangle?

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| **Prioritization** |
| Skill is assessed in the general assessment  Skill is assessed in the alternate assessment  Skill is required for future learning in the content area  Needed in next age/appropriate environment  Required for instructional activities in a variety of practice communities  Lesson plan available in SAS Voluntary Model Curriculum |

**Example of General Education Instructional Activity:**

Students will:

* Identify and use properties of congruent and similar polygons

**Opener: The teacher will pass out white boards, markers, and erasers. If white boards are not available, the teacher can pass out white paper and markers. Each student may need up to 3-5 sheets each.**

**Review Vocabulary: Adjacent angles, Adjacent sides, Corresponding angles, Corresponding sides, and Congruent figures**

**1. The teacher will direct the students to sketch any triangle. The teacher will then ask the students to mark and label an example of adjacent angles. When all the students have completed this task, the teacher will ask the students to hold up their sketches. The teacher will monitor the students' sketches for understanding. The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check. After the students have had an opportunity to compare their sketches with a correct example, the teacher will ask if further sketches and/or explanations are needed. It may be necessary to draw several examples on the board.**

**2. Using the same sketch (or a new one if the original is too cluttered with markings), the teacher will ask the students to mark and label an example of adjacent sides. When all the students have completed this task, the teacher will ask the students to hold up their sketches. The teacher will monitor the students' sketches for understanding. The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check. After the students have had an opportunity to compare their sketches with a correct example, the teacher will ask if further sketches and/or explanations are needed. It may be necessary to draw several examples on the board.**

**3. The teacher will now direct the students to sketch two identical triangles. The teacher will then ask the students to mark and label an example of corresponding angles. When all the students have completed this task, the teacher will ask the students to hold up their sketches. The teacher will monitor the students' sketches for understanding. The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check. After the students have had an opportunity to compare their sketches with a correct example, the teacher will ask if further sketches and/or explanations are needed. It may be necessary to draw several examples on the board.**

**4. Using the same sketch (or a new one if the original is too cluttered with markings), the teacher will ask the students to mark and label an example of corresponding sides. When all the students have completed this task, the teacher will ask the students to hold up their sketches. The teacher will monitor the students' sketches for understanding. The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check. After the students have had an opportunity to compare their sketches with a correct example, the teacher will ask if further sketches and/or explanations are needed. It may be necessary to draw several examples on the board.**

**5. The teacher will now direct the students to sketch an example of congruent figures. When all the students have completed this task, the teacher will ask the students to hold up their sketches. The teacher will monitor the students' sketches for understanding. The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check. After the students have had an opportunity to compare their sketches with a correct example, the teacher will ask if further sketches and/or explanations are needed. It may be necessary to draw several examples on the board.**

**6. The Teacher will build on the prior knowledge by asking the students the following series of questions:**

**The students will write the numbers 1-5 down the left-hand side of the white board (or paper) and answer each question.**

**The teacher will ask a question, wait for the student to write a response to the question, and after all students have written an answer, the students will hold up their answers.**

**Question #1: How many pairs of corresponding, congruent angles can be identified in two congruent triangles?**

**(Ans: 3 pairs of angles)**

**The teacher will monitor the responses for accuracy. The teacher will sketch a pair of congruent triangles on the board and mark all the pairs of corresponding, congruent angles. The students will be given an opportunity to self-check. After the students see the correct response, the teacher will ask if further sketches and/or explanations are needed. It may be necessary to draw several examples on the board.**

**Question #2: How many pairs of corresponding, congruent sides can be identified in two congruent triangles?**

**(Ans: 3 pairs of sides)**

**The teacher will monitor the responses for accuracy. The teacher will sketch a pair of congruent triangles on the board and mark all the pairs of corresponding, congruent sides. The students will be given an opportunity to self-check. After the students see the correct response, the teacher will ask if further sketches and/or explanations are needed. It may be necessary to draw several examples on the board.**

**Question #3: How many total pairs of corresponding, congruent parts can be identified in two congruent triangles?**

**(Ans: 3 pairs of angles and 3 pairs of sides for a total of 6 pairs of corresponding, congruent parts)**

**The teacher will monitor the responses for accuracy. The teacher will sketch a pair of congruent triangles on the board and mark all the pairs of corresponding, congruent parts. The students will be given an opportunity to self-check. After the students see the correct response, the teacher will ask if further sketches and/or explanations are needed. It may be necessary to draw several examples on the board.**

**Question #4: Using the definition of congruent triangles, how many of the corresponding, congruent parts are necessary to identify two triangles congruent? (The teacher will emphasize the need to satisfy the definition of congruent triangles.)**

**(Ans: to use the definition to prove two triangles congruent, you must show all 6 pairs of corresponding parts congruent.)**

**The teacher will monitor the responses for accuracy. The teacher will refer back to the sketch of the congruent triangles on the board with all the pairs of corresponding, congruent parts marked congruent. The students will be given an opportunity to self-check. After the students see the correct response, the teacher will read the definition of congruent triangles to the class. The teacher will ask if further sketches and/or explanations are needed. It may be necessary to draw several examples on the board.**

**Question #5: YES or NO: Do you think fewer parts can be used to guarantee a unique triangle?**

**(Ans: answers will vary)**

**The teacher will monitor the responses for accuracy. The teacher will not indicate which responses are correct, but will have students share their thoughts with their classmates. Ask for volunteers or choose at least 5 or 6 students for this activity.**

Pass out worksheet:  Investigating Congruent Triangles

[**M:\Worksheet Investigating Congruence Theorems.docx**](http://websites.pdesas.org/shirlann_finch/2010/3/22/30296/file.aspx)

At this time, give students the opportunity to answer **Question #1** on the worksheet. (2-3 minutes)

Encourage the students to write their thoughts as well as identifying some new ideas they heard from their classmates.

**7.** At this time, the teacher will direct the students to the Congruence Theorems interactive software found on the Thinkfinity website and open up the same website in the front of the room for demonstration purposes.  If a classroom set of computers is not available, the teacher can use the computer in the front of the room and the students can follow the teacher through the website.

[**Congruence Theorem Software**](http://websites.pdesas.org/shirlann_finch/2010/3/17/27762/content.aspx)

**The teacher will direct the students to open the Instructions and Exploration menus to locate the instructions for working through and navigating the software. The teacher will proceed through the same steps on the computer in the front of the room.**

**The students will read the Instructions to themselves as the teacher reads out loud.**

**Allow 5-10 minutes for the teacher and the students to read through the directions and allow the students to explore the software. As the students are investigating the necessary skills to manipulate the parts of a triangle, the teacher should be walking around the room and giving appropriate suggestions when a student appears frustrated. After the students have had an opportunity to become familiar with constructing a triangle, ask the students if they have any questions. It may be necessary for the teacher to demonstrate certain skills or help some of the students individually. This is also an excellent opportunity for students with more computer experience to help their peers.**

**8. After free-exploration, the teacher will instruct the students to scroll down until only the Exploration section and the interactive screen are on their monitors. The teacher will proceed through the same step on the computer in the front of the room.**

**The students and the teacher will now work together through the sample problem under the Exploration heading. After each new step, give students ample time to complete the action on their individual computers. Do not move on until all the students have given a "thumbs up!"**

**At the end of the exploration, there is a question: "Given two triangles for which two sides and an adjacent angle are congruent, are the triangles congruent? Explain." Ask for volunteers to share their answers.**

**9. Next, as you demonstrate in the front of the room, have the students move one triangle so it lies on top of the other triangle. The teacher will ask the students if all the corresponding parts of the two triangles are congruent. (Yes). The teacher will continue with the question does this fit the definition of congruent triangles? (Yes). To complete the sample exploration, the teacher will read the question at the bottom of the monitor: "Are these triangles congruent?" click YES and have the students do the same.**

**The students will now click the Reset button and determine if they always get the same results. (With this combination they will always get the same result.)**

**10. The students will now return to the worksheet and read Question #2 as the teacher reads out loud. Using the results from the sample exploration, the teacher will illustrate on the board the appropriate answers in each column for "1)” on the chart.**

**Name the parts you checked on the software Is the second triangle congruent to the first? Can you make a triangle that is not congruent with the original?**

|  |  |  |
| --- | --- | --- |
| Name the parts you checked on the software | Is the second triangle congruent to the first? | Can you make a triangle that is not congruent with the original? |
| 1)   side, side, angle | YES | No |
| 2) |  |  |

**The students are now ready to work through the interactive software on their own and record their results on the worksheet.**

**Allow students 15-20 minutes to work independently.**

**11. Next, the teacher will organize the students into small groups (a maximum of 4 students is recommended). The students will compare their results and answer Question #3. (5 minutes)**

**At the end of class, have each group share a conclusion (successful and unsuccessful). The teacher will record (on chalk board, white board, or large post-it-note) the successful combinations of triangle parts the students discovered were needed to make a unique triangle. Continue until all the following combinations have been identified. It may be necessary to demonstrate examples that were not discovered. Have students write the successful combinations for the answer to Question #4.**

**1) Side-Side-Side (SSS)**

**2) Side-Angle-Side (SAS)**

**3) Angle-Side-Angle (ASA)**

**4) Angle-Angle-Side (AAS)**

**5) Hypotenuse-Leg (HL-the special case of Angle-Side-Side!)**

**Note: the students may not discover Hypotenuse-Leg on their own**

**The students can now answer Question #5 and finish the worksheet. Collect the worksheets.**

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| **General Instructional Format** | **Formative Assessment Options** |
| **Cooperative learning**  **Project based**  **Performance event/task**  **Note-taking**  **Presentation**  **Direct Instruction (I do, We do, You do)**  **Indirect Instruction**  **Other:** **graphic organizers** | **Observation with Data Collection**  **Random Reporter**  **Ticket out the door**  **Think Pair Share**  **Student work sample**  **Video tape**  **Multiple choice Item**  **Open response Item**  **Item Other:** |

**Access to the Instructional Activity for Students at Different Communication Levels**

**Symbolic**

Students will:

* Identify and use properties of congruent and similar polygons

**Opener: The teacher will pass out white boards, markers, and erasers. If white boards are not available, the teacher can pass out white paper and markers. Each student may need up to 3-5 sheets each.**

**Review Vocabulary: Adjacent angles, Adjacent sides, Corresponding angles, Corresponding sides, and Congruent figures**

**Give students a graphic organizer populated with the targeted vocabulary words.**

**1.** The teacher will direct the students to sketch any triangle.  The teacher will ask them to divide the triangle into two triangles. The teacher will then ask the students to mark and label an example of adjacent angles ( ).  When all the students have completed this task, the teacher will ask the students to hold up their sketches.  The teacher will monitor the students' sketches for understanding.  The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check.  After the students have had an opportunity to compare their sketches with a correct example, and copy this example onto their graphic organizer in the section labeled adjacent angles. The teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**2.** Using the same sketch (or a new one if the original is too cluttered with markings), the teacher will ask the students to mark and label an example of adjacent sides ( ).  When all the students have completed this task, the teacher will ask the students to hold up their sketches.  The teacher will monitor the students' sketches for understanding.  The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check.  After the students have had an opportunity to compare their sketches with a correct example, and copy this example onto their graphic organizer labeled adjacent sides, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**3.** The teacher will now direct the students to sketch two identical triangles ( ).  The teacher will then ask the students to mark and label an example of corresponding angles ( ).  When all the students have completed this task, the teacher will ask the students to hold up their sketches.  The teacher will monitor the students' sketches for understanding.  The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check.  After the students have had an opportunity to compare their sketches with a correct example, and copy this example onto their graphic organizer labeled corresponding angles. The teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**4.** Using the same sketch (or a new one if the original is too cluttered with markings), the teacher will ask the students to mark and label an example of corresponding sides ( ).  When all the students have completed this task, the teacher will ask the students to hold up their sketches.  The teacher will monitor the students' sketches for understanding.  The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check.  After the students have had an opportunity to compare their sketches with a correct example, and copy this example onto their graphic organizer labeled corresponding sides. The teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**5.** The teacher will now direct the students to sketch an example of congruent figures.  When all the students have completed this task, the teacher will ask the students to hold up their sketches.  The teacher will monitor the students' sketches for understanding.  The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check.  After the students have had an opportunity to compare their sketches with a correct example, and copy this example onto their graphic organizer, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**6.** The Teacher will build on the prior knowledge by asking the students the following series of questions:

The students will write the numbers 1-5 down the left-hand side of the white board (or paper) and answer each question.  Some students may be provided the following questions. The teacher will ask a question, wait for the student to write a response to the question, and after all students have written an answer, the students will hold up their answers. Students should be reminded to refer to their graphic organizer when answering the questions.

**Question #1: How many pairs of corresponding, congruent angles can be identified in two congruent triangles ?**

**(Ans: 3 pairs of angles)**

**The teacher will monitor the responses for accuracy. The teacher will sketch a pair of congruent triangles on the board and mark all the pairs of corresponding, congruent angles. The students will be given an opportunity to self-check. After the students see the correct response and record the responses in their graphic organizer, the teacher will ask if further sketches and/or explanations are needed. It may be necessary to draw several examples on the board.**

**Question #2: How many pairs of corresponding, congruent sides can be identified in two congruent triangles ?**

**(Ans: 3 pairs of sides)**

**The teacher will monitor the responses for accuracy. The teacher will sketch a pair of congruent triangles on the board and mark all the pairs of corresponding, congruent sides. The students will be given an opportunity to self-check. After the students see the correct response and record the responses in their graphic organizer, the teacher will ask if further sketches and/or explanations are needed. It may be necessary to draw several examples on the board.**

**Question #3: How many total pairs of corresponding, congruent parts can be identified in two congruent triangles ?**

**(Ans: 3 pairs of angles and 3 pairs of sides for a total of 6 pairs of corresponding, congruent parts)**

**The teacher will monitor the responses for accuracy. The teacher will sketch a pair of congruent triangles on the board and mark all the pairs of corresponding, congruent parts. The students will be given an opportunity to self-check. After the students see the correct response and record the responses in their graphic organizer, the teacher will ask if further sketches and/or explanations are needed. It may be necessary to draw several examples on the board.**

**Question #4: Using the definition of congruent triangles, how many of the corresponding, congruent parts are necessary to identify two triangles congruent? (The teacher will emphasize the need to satisfy the definition of congruent triangles.)**

**(Ans: to use the definition to prove two triangles congruent, you must show all 6 pairs of corresponding parts congruent.)**

**The teacher will monitor the responses for accuracy. The teacher will refer back to the sketch of the congruent triangles on the board with all the pairs of corresponding, congruent parts marked congruent. The students will be given an opportunity to self-check. After the students see the correct response, the teacher will read the definition of congruent triangles to the class. The teacher will ask if further sketches and/or explanations are needed. It may be necessary to draw several examples on the board.**

**Question#5: YES or NO: Do you think fewer parts can be used to guarantee a unique triangle?**

**(Ans: answers will vary)**

**The teacher will monitor the responses for accuracy. The teacher will not indicate which responses are correct, but will have students share their thoughts with their classmates. Ask for volunteers or choose at least 5 or 6 students for this activity.**

**Pass out worksheet: Investigating Congruent Triangles**

[**M:\Worksheet Investigating Congruence Theorems.docx**](http://websites.pdesas.org/shirlann_finch/2010/3/22/30296/file.aspx)

**At this time, give students the opportunity to answer Question #1 on the worksheet. (2-3 minutes)**

**Encourage the students to write their thoughts as well as identifying some new ideas they heard from their classmates.**

**7. At this time, the teacher will direct the students to the Congruence Theorems interactive software found on the Thinkfinity website and open up the same website in the front of the room for demonstration purposes. If a classroom set of computers is not available, the teacher can use the computer in the front of the room and the students can follow the teacher through the website.**

[**Congruence Theorem Software**](http://websites.pdesas.org/shirlann_finch/2010/3/17/27762/content.aspx)

**The teacher will direct the students to open the Instructions and Exploration menus to locate the instructions for working through and navigating the software. Students may be provided task analyzed steps for working with the software. The teacher will proceed through the same steps on the computer in the front of the room.**

**The students will read the Instructions to themselves as the teacher reads out loud. Allow 5-10 minutes for the teacher and the students to read through the directions and allow the students to explore the software. As the students are investigating the necessary skills to manipulate the parts of a triangle, the teacher should be walking around the room and giving appropriate suggestions when a student appears frustrated. After the students have had an opportunity to become familiar with constructing a triangle, ask the students if they have any questions. It may be necessary for the teacher to demonstrate certain skills or help some of the students individually. This is also an excellent opportunity for students with more computer experience to help their peers.**

**8. After free-exploration, the teacher will instruct the students to scroll down until only the Exploration section and the interactive screen are on their monitors. The teacher will proceed through the same step on the computer in the front of the room.**

**The students and the teacher will now work together through the sample problem under the Exploration heading. After each new step, give students ample time to complete the action on their individual computers. Do not move on until all the students have given a "thumbs up!"**

**At the end of the exploration, there is a question: "Given two triangles for which two sides and an adjacent angle are congruent, are the triangles congruent? Explain." Ask for volunteers to share their answers. You may want to give this question to the students at the start of their exploration so that they may think about it as they explore the program.**

**9. Next, as you demonstrate in the front of the room, have the students move one triangle so it lies on top of the other triangle. Students might work in pairs to compare triangles. The teacher will ask students who remembers the definition of congruent triangles. Students may refer to their graphic organizer of vocabulary words. The teacher will ask the students if all the corresponding parts of the two triangles are congruent. (Yes). The teacher will continue with the question does this fit the definition of congruent triangles? (Yes). To complete the sample exploration, the teacher will read the question at the bottom of the monitor: "Are these triangles congruent?" click YES and have the students do the same.**

**The students will now click the Reset button and determine if they always get the same results. (With this combination they will always get the same result.)**

**10. The students will now return to the worksheet and read Question #2 as the teacher reads out loud. Using the results from the sample exploration, the teacher will illustrate on the board the appropriate answers in each column for "1)” on the chart.**

**Name the parts you checked on the software Is the second triangle congruent to the first? Can you make a triangle that is not congruent with the original? Which parts are corresponding?**

|  |  |  |  |
| --- | --- | --- | --- |
| Name the parts you checked on the software | Is the second triangle congruent to the first? | Can you make a triangle that is not congruent with the original? | Which parts are corresponding? |
| 1)   side, side, angle or AC, AB,  <A |  |  |  |
| 2) side, side, angle or TZ, TR,  <T | YES | No | AC & TZ; AB & TR;  <A & <T |

**The students are now ready to work through the interactive software on their own and record their results on the worksheet.**

**Allow students 15-20 minutes to work independently or in pairs.**

**Give students the following combinations to explore and record observations on their worksheet.**

     1)  Side-Side-Side          (SSS)

     2)  Side-Angle-Side        (SAS)

     3)  Angle-Side-Angle       (ASA)

     4)  Angle-Angle-Side       (AAS)

**11. Next, the teacher will organize the students into small groups (a maximum of 4 students is recommended). The students will compare their results and answer Question #3. (5 minutes)**

**At the end of class, have each group share a conclusion (successful and unsuccessful). The teacher will record (on chalk board, white board, or large post-it-note) the successful combinations of triangle parts the students discovered were needed to make unique triangles. Continue until all the following combinations have been identified. It may be necessary to demonstrate examples that were not discovered. Have students write the successful combinations for the answer to Question #4.**

**1) Side-Side-Side (SSS)**

**2) Side-Angle-Side (SAS)**

**3) Angle-Side-Angle (ASA)**

**4) Angle-Angle-Side (AAS)**

**5) Hypotenuse-Leg (HL-the special case of Angle-Side-Side!)**

**Note: The students may not discover Hypotenuse-Leg on their own. The students can now answer Question #5 and finish the worksheet. Collect the worksheets.**

**Emerging Symbolic**

Students will:

* Identify and use properties of congruent and similar polygons

**Opener: The teacher will pass out white boards, markers, and erasers. If white boards are not available, the teacher can pass out white paper and markers. Each student may need up to 3-5 sheets each.**

Review Vocabulary:  Adjacent angles, Adjacent sides, Corresponding angles, Corresponding sides, Congruent triangles, and Congruent figures. Give students a graphic organizer populated with the targeted vocabulary words and corresponding picture representations.

The teacher will use a model/picture representation of vocabulary when presenting it to the class for review. Student will identify corresponding vocabulary by indicating the appropriate one on the graphic organizer.

**1.** The teacher will direct the students to sketch or trace any triangle Tracing can be done by using a stencil or by using a piece of transparent paper over a triangular shape.  Given a visual representation of a triangle already divided, the teacher will ask them to divide the triangle into two triangles . The student may divide the triangle using a pen and straight edge, pencil, dobber, wiki sticks, etc. The teacher will then ask the students to mark and label the adjacent angles .  Provide the student with a picture, object or tactile representation for adjacent angles to add to the vocabulary graphic organizer and use as a model to label adjacent angless on the triangle. The student may mark the adjacent angles with pen, pencil, dobber, wiki sticks, etc. When all the students have completed this task, the teacher will ask the students to hold up their sketches.  The teacher will monitor the students' sketches for understanding.  The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check.  After the students have had an opportunity to compare their sketches with a correct example, and copy or paste the drawing/tracing they just created onto their graphic organizer, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**2.** Using the same sketch (or a new one if the original is too cluttered with markings), the teacher will ask the students to mark and label an example of adjacent sides .  Provide the student with a picture, object or tactile representation for adjacent sides to add to the vocabulary graphic organizer and use as a model to label adjacent sides on the triangle. The student may mark the adjacent sides with pen, pencil, dobber, wiki sticks, etc. When all the students have completed this task, the teacher will ask the students to hold up their sketches.  The teacher will monitor the students' sketches for understanding.  The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check.  After the students have had an opportunity to compare their sketches with a correct example, and copy or paste the drawing/tracing/tactile representation they just created onto their graphic organizer, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**3.** The teacher will now direct the students to sketch two identical triangles ( ).  Provide the student with a picture representation of identical triangles to add to the vocabulary graphic organizer and use as a model. The teacher will then ask the students to mark and label an example of corresponding angles ( ).  Provide the student with a picture, object or tactile representation for corresponding angles to add to the vocabulary graphic organizer and use as a model to label corresponding angles on the triangle. The student may mark the corresponding angles with pen, pencil, dobber, wiki sticks, etc. When all the students have completed this task, the teacher will ask the students to hold up their sketches.  The teacher will monitor the students' sketches for understanding.  The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check.  After the students have had an opportunity to compare their sketches with a correct example, and copy or paste the drawing/tracing/tactile representation they just created onto their graphic organizer, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**4.** Using the same sketch (or a new one if the original is too cluttered with markings), the teacher will ask the students to mark and label an example of corresponding sides ( ).  Provide the student with a picture, object or tactile representation for corresponding sides to add to the vocabulary graphic organizer and use as a model to label corresponding sides on the triangle. The student may mark the corresponding sides with pen, pencil, dobber, wiki sticks, etc. When all the students have completed this task, the teacher will ask the students to hold up their sketches.  The teacher will monitor the students' sketches for understanding.  The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check.  After the students have had an opportunity to compare their sketches with a correct example, and copy or paste the drawing/tracing/tactile representation they just created onto their graphic organizer, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**5.** The teacher will now direct the students to sketch an example of congruent figures ( ). Provide the student with a picture representation of congruent triangles to add to the vocabulary graphic organizer and use as a model.  Have student compare the identical triangles with the congruent triangles and make some connections. When all the students have completed this task, the teacher will ask the students to hold up their sketches.  The teacher will monitor the students' sketches for understanding.  The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check.  After the students have had an opportunity to compare their sketches with a correct example, and copy or paste the drawing/tracing/tactile representation they just created onto their graphic organizer, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**6.** The Teacher will build on the prior knowledge by asking the students the following series of questions:

Give the student a piece of paper with the numbers 1-5 written down the left-hand side and explain that they will be answering each question.  Provide the student with the written questions paired with picture representations.

The teacher will ask a question, wait for the student to circle a response to the question, and after all students have written an answer, the students will hold up their answers.

**Question #1**:  How many pairs of corresponding, congruent angles ( ) can be identified in two congruent triangles?

The teacher will ask the student to identify two congruent triangles on their graphic organizer. Then the teacher will ask the student to identify the pairs of corresponding congruent angles. Ask the student to count the pairs of congruent angles and indicate that on the answer sheet choosing the correct number from a choice of three.

(Ans: 3 pairs of angles)

The teacher will monitor the responses for accuracy.  The teacher will sketch a pair of congruent triangles on the board and mark all the pairs of corresponding, congruent angles.  The students will be given an opportunity to self-check.  After the students see the correct response, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**Question #2:**  How many pairs of corresponding, congruent sides ( ) can be identified in two congruent triangles?

The teacher will ask the student to identify two congruent triangles on their graphic organizer. Then the teacher will ask the student to identify the pairs of corresponding congruent sides. Ask the student to count the pairs of congruent sides and indicate that on the answer sheet/white board from a choice of three.

          (Ans: 3 pairs of sides)

The teacher will monitor the responses for accuracy.  The teacher will sketch a pair of congruent triangles on the board and mark all the pairs of corresponding, congruent sides.  The students will be given an opportunity to self-check.  After the students see the correct response, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**Question #3:**  How many total pairs of corresponding, congruent parts ( ) can be identified in two congruent triangles?

Ask the student how many pairs of congruent angles did they count in question #1? **\_\_\_\_\_\_**

(The student must indicate this from their answer to Question 1)

Ask the student how many pairs of congruent sides did they count in question #2? **\_\_\_\_\_\_**

(The student must indicate this from their answer to Question 2)

Show the student that if we add these together we get the total number of congruent parts.

How many pairs of congruent parts do we have? Answer **\_\_\_\_\_\_\_\_\_**

Have the student indicate the answer on the answer sheet/white board.

(Ans:  3 pairs of angles and 3 pairs of sides for a total of 6 pairs of corresponding, congruent parts)

The teacher will monitor the responses for accuracy.  The teacher will sketch a pair of congruent triangles on the board and mark all the pairs of corresponding, congruent parts.  The students will be given an opportunity to self-check.  After the students see the correct response, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**Question #4**:  YES or NO: Using the definition of congruent triangles, must you have six corresponding, congruent pairs to identify two triangles as congruent?  (The teacher will emphasize the need to satisfy the definition of congruent triangles.)

          (Ans:  The answer is Yes)

The teacher will monitor the responses for accuracy.  The teacher will refer back to the sketch of the congruent triangles on the board with all the pairs of corresponding, congruent parts marked congruent.  The students will be given an opportunity to self-check.  After the students see the correct response, the teacher will read the definition of congruent triangles to the class.  The teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**Question#5:**  YES  or  NO:  Do you think fewer parts can be used to guarantee a unique triangle?

                    (Ans:  answers will vary)

The teacher will monitor the responses for accuracy.  The teacher will not indicate which responses are correct, but will have students share their thoughts with their classmates.  Ask for volunteers or choose at least 5 or 6 students for this activity.

Pass out worksheet:  Investigating Congruent Triangles

[**M:\Worksheet Investigating Congruence Theorems.docx**](http://websites.pdesas.org/shirlann_finch/2010/3/22/30296/file.aspx)

At this time, give students the opportunity to answer **Question #1** on the worksheet. (2-3 minutes)

Encourage the students to write their thoughts as well as identifying some new ideas they heard from their classmates.

7. At this time, the teacher will direct the students to the Congruence Theorems interactive software found on the Thinkfinity website and open up the same website in the front of the room for demonstration purposes.  If a classroom set of computers is not available, the teacher can use the computer in the front of the room and the students can follow the teacher through the website.

[**Congruence Theorem Software**](http://websites.pdesas.org/shirlann_finch/2010/3/17/27762/content.aspx)

**The teacher will direct the students to open the Instructions and Exploration menus to locate the instructions for working through and navigating the software. Students may be provided task analyzed steps paired with picture representations for working with the software. The teacher will proceed through the same steps on the computer in the front of the room.**

The students will read the Instructions to themselves as the teacher reads out loud. Allow 5-10 minutes for the teacher and the students to read through the directions and allow the students to explore the software.  It may be necessary to find a similar program that allows for switch access or to create a similar program using assistive technology programs such as IntelliSuites. As the students are investigating the necessary skills to manipulate the parts of a triangle, the teacher should be walking around the room and giving appropriate suggestions when a student appears frustrated.  After the students have had an opportunity to become familiar with constructing a triangle, ask the students if they have any questions.  It may be necessary for the teacher to demonstrate certain skills or help some of the students individually. This is also an excellent opportunity for students with more computer experience to help their peers.

**8.** After free-exploration, the teacher will instruct the students to scroll down until only the Exploration section and the interactive screen are on their monitors.  The teacher will proceed through the same step on the computer in the front of the room.

The students and the teacher will now work together through the sample problem under the Exploration heading. After each new step, give students ample time to complete the action on their individual computers. Do not move on until all the students have given a "thumbs up!"

At the end of the exploration, there is a question:  "Given two triangles for which two sides and an adjacent angle are congruent, are the triangles congruent?  Explain."  Ask for volunteers to share their answers.  **You may want to give this question paired with picture representations to the students at the start of their exploration so that they may think about it as they explore the program.**

**9**. Next, as you demonstrate in the front of the room, have the students move one triangle so it lies on top of the other triangle. **Students might work in pairs to compare triangles. Some students may need manipulatives to compare the congruent triangles. The teacher will ask students “who remembers the definition of congruent triangles?”. Students may refer to their graphic organizer of vocabulary words.** The teacher will ask the students if all the corresponding parts of the two triangles are congruent. (Yes).  The teacher will continue with the question does this fit the definition of congruent triangles? (Yes).  To complete the sample exploration, the teacher will read the question at the bottom of the monitor:  "Are these triangles congruent?" click YES and have the students do the same. If the students are using manipulatives to determine if pairs of triangles are congruent, they can sort the pairs in a T-chart labeled as Yes and No or Congruent/Not Congruent.

The students will now click the Reset button and determine if they always get the same results.  (With this combination they will always get the same result.)

**10.** The students will now return to the worksheet and read **Question #2** as the teacher reads out loud.  Using the results from the sample exploration, the teacher will illustrate on the board the appropriate answers in each column for "1” on the chart. Students may need to include picture representations with sides and angles marked based on each scenario presented in column “1” of the chart.

|  |  |  |  |
| --- | --- | --- | --- |
| Name the parts you checked on the software | Is the second triangle congruent to the first? | Can you make a triangle that is not congruent with the original? | Which parts are corresponding? |
| side, side, angle or AC, AB,  <A |  |  |  |
| side, side, angle or TZ, TR,  <T | YES | No | AC & TZ; AB & TR;  <A & <T |

The students are now ready to work through the interactive software on their own and record their results on the worksheet.

Allow students 15-20 minutes to work independently or in pairs.

**Give students the following combinations to explore and record observations on their worksheet.**

     1)  Side-Side-Side          (SSS)

     2)  Side-Angle-Side        (SAS)

     3)  Angle-Side-Angle       (ASA)

     4)  Angle-Angle-Side       (AAS)

**11.** Next, the teacher will organize the students into small groups (a maximum of 4 students is recommended).  The students will compare their results and answer **Question #3**. (5 minutes)

At the end of class, have each group share a conclusion (successful and unsuccessful).  The teacher will record (on chalk board, white board, or large post-it-note) the successful combinations of triangle parts the students discovered were needed to make unique triangles.  Continue until all the following combinations have been identified.  It may be necessary to demonstrate examples that were not discovered.  Have students write the successful combinations for the answer to **Question #4**.

     1)  Side-Side-Side          (SSS)

     2)  Side-Angle-Side        (SAS)

     3)  Angle-Side-Angle       (ASA)

     4)  Angle-Angle-Side       (AAS)

     5)  Hypotenuse-Leg        (HL-the special case of Angle-Side-Side!)

Note:  The students may not discover Hypotenuse-Leg on their own. The students can now answer **Question #5** and finish the worksheet. Collect the worksheets.

**Pre-Symbolic**

Students will:

* Identify and use properties of congruent and similar polygons

**Opener: The teacher will pass out white boards, markers, and erasers. If white boards are not available, the teacher can pass out white paper and markers. Each student may need up to 3-5 sheets each.**

Review Vocabulary:  Adjacent angles, Adjacent sides, Corresponding angles, Corresponding sides, Congruent triangles, and Congruent figures

Give students the following targeted vocabulary words and corresponding object or tactile representation paired with a picture representation:

1. Triangle
2. Angles
3. Sides
4. Adjacent
5. Corresponding
6. Congruent/same

The teacher will present the object or tactile representation paired with a picture representation of each vocabulary word as it is introduced to the class. Student will keep the vocabulary words in a graphic organizer, in a basket, etc. for easy access throughout the lesson.

**1**. The teacher will direct the students to sketch or trace or obtain any triangle. Tracing can be done by using a stencil or by using a piece of transparent paper over a triangular shape or the student may choose a triangle by eye gaze, reach and grasp or manipulating on a computer screen.  Given a visual and tactile representation of a triangle already divided, the teacher will ask them to divide the triangle into two triangles . The student may divide the triangle using a pen and straight edge, pencil, dobber, wiki sticks, etc., or choose the correct picture or tactile representation of a triangle divided. The teacher will then ask the students to mark and label the adjacent angles   . Provide the object/tactile vocabulary representations of adjacent and angles to the student and have the student identify a picture, model or object triangle which has all the adjacent angles labeled. When all the students have completed this task, the teacher will ask the students to hold up their sketches.  The teacher will monitor the students' sketches for understanding.  The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check.  After the students have had an opportunity to compare their sketches with a correct example and copy this example onto a graphic organizer or place it in a basket or other type of collection, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**2.** Using the same sketch (or a new one if the original is too cluttered with markings), the teacher will ask the students to mark and label an example of adjacent sides ( ).  Provide the object/tactile vocabulary representations of adjacent and sides to the student and have the student identify a picture, model or object triangle which has all the adjacent sides labeled. When all the students have completed this task, the teacher will ask the students to hold up their sketches.  The teacher will monitor the students' sketches for understanding.  The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check.  After the students have had an opportunity to compare their sketches with a correct example, and copy this example onto a graphic organizer or place it in a basket or other type of collection, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**3**. The teacher will now direct the students to sketch two identical triangles ( ).  Provide the object/tactile vocabulary representations of same (congruent) and triangle to the student and have the student identify a picture, model or objects of two triangles that are exactly the same. The teacher will then ask the students to mark and label an example of corresponding angles ( ).  Provide the object/tactile vocabulary representations of corresponding and angles to the student and have the student identify a picture, model or objects of two triangles which have adjacent angles labeled. When all the students have completed this task, the teacher will ask the students to hold up their sketches.  The teacher will monitor the students' sketches for understanding.  The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check.  After the students have had an opportunity to compare their sketches with a correct example, and copy this example onto a graphic organizer or place it in a basket or other type of collection, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**4**. Using the same sketch (or a new one if the original is too cluttered with markings), the teacher will ask the students to mark and label an example of corresponding sides ( ).  Provide the object/tactile vocabulary representations of corresponding and sides to the student and have the student identify a picture, model or objects of two triangles which have adjacent sides labeled. When all the students have completed this task, the teacher will ask the students to hold up their sketches.  The teacher will monitor the students' sketches for understanding.  The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check.  After the students have had an opportunity to compare their sketches with a correct example, and copy this example onto a graphic organizer or place it in a basket or other type of collection, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**5.** The teacher will now direct the students to sketch an example of congruent figures ( ).  Provide the object/tactile vocabulary representations of congruent (same) and triangle to the student and have the student identify a picture, model or objects of two triangles that are congruent. When all the students have completed this task, the teacher will ask the students to hold up their sketches.  The teacher will monitor the students' sketches for understanding.  The teacher will choose a student to draw their sketch on the classroom board so all the students can compare their individual sketches and self-check.  After the students have had an opportunity to compare their sketches with a correct example, and copy this example onto a graphic organizer or place it in a basket or other type of collection, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**6.** The Teacher will build on the prior knowledge by asking the students the following series of questions:

Give the student 5 baskets labeled with the tactile and picture representation of each question on it.

The teacher will ask a question, wait for the student to choose a response to the question and place it in the basket, and after all students have written an answer, the students will hold up their answers.

**Question #1:**  How many pairs of corresponding, congruent angles ( ) can be identified in two congruent triangles?

The teacher will ask the student to identify two congruent triangles by choosing the correct picture, object or tactile representation. Each corresponding angle should be labeled ( ) Then the teacher will ask the student to identify how many pairs of corresponding congruent angles there are. The student may identify the pairs of corresponding angles by by eye gazing a pair and hitting the switch to count each time a pair is identified, using an adaptive program and hitting a switch each time a pair is highlighted, etc. The student’s answer would then be placed in the basket representing question number one.

(Ans: 3 pairs of angles)

The teacher will monitor the responses for accuracy.  The teacher will sketch a pair of congruent triangles on the board and mark all the pairs of corresponding, congruent angles.  The students will be given an opportunity to self-check.  After the students see the correct response, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**Question #2:**  How many pairs of corresponding, congruent sides ( ) can be identified in two congruent triangles?

The teacher will ask the student to identify two congruent triangles by choosing the correct picture, object or tactile representation. Each corresponding side should be labeled ( ) Then the teacher will ask the student to identify how many pairs of corresponding congruent sides there are. The student may identify the pairs of corresponding sides by by eye gazing a pair and hitting the switch to count each time a pair is identified, using an adaptive program and hitting a switch each time a pair is highlighted, etc. The student’s answer would then be placed in the basket representing question number two.

(Ans: 3 pairs of sides)

The teacher will monitor the responses for accuracy.  The teacher will sketch a pair of congruent triangles on the board and mark all the pairs of corresponding, congruent sides.  The students will be given an opportunity to self-check.  After the students see the correct response, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**Question #3**:  How many total pairs of corresponding, congruent parts ( ) can be identified in two congruent triangles?

Ask the student how many pairs of congruent angles did they count in question #1? **\_\_\_\_\_\_**

(The student must indicate this by collecting the answer from their Question 1 basket)

Ask the student how many pairs of congruent sides did they count in question #2? **\_\_\_\_\_\_**

(The student must indicate this by collecting the answer from their Question 2 basket)

Show the student that if we add these together we get the total number of congruent parts.

How many pairs of congruent parts do we have? Answer **\_\_\_\_\_\_\_\_\_**

The student’s answer would then be placed in the basket representing question number three.

          (Ans:  3 pairs of angles and 3 pairs of sides for a total of 6 pairs of corresponding, congruent parts)

The teacher will monitor the responses for accuracy.  The teacher will sketch a pair of congruent triangles on the board and mark all the pairs of corresponding, congruent parts.  The students will be given an opportunity to self-check.  After the students see the correct response, the teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**Question #4:**  YES or NO: Using the definition of congruent triangles, must you have six corresponding, congruent pairs to identify two triangles as congruent?  (The teacher will emphasize the need to satisfy the definition of congruent triangles.)

(Ans:  The answer is Yes)

The teacher will monitor the responses for accuracy.  The teacher will refer back to the sketch of the congruent triangles on the board with all the pairs of corresponding, congruent parts marked congruent.  The students will be given an opportunity to self-check.  After the students see the correct response, the teacher will read the definition of congruent triangles to the class.  The teacher will ask if further sketches and/or explanations are needed.  It may be necessary to draw several examples on the board.

**Question#5:**  YES  or  NO:  Do you think fewer parts can be used to guarantee a unique triangle?

(Ans:  answers will vary)

The teacher will monitor the responses for accuracy.  The teacher will not indicate which responses are correct, but will have students share their thoughts with their classmates.  Ask for volunteers or choose at least 5 or 6 students for this activity.

Pass out worksheet:  Investigating Congruent Triangles

[**M:\Worksheet Investigating Congruence Theorems.docx**](http://websites.pdesas.org/shirlann_finch/2010/3/22/30296/file.aspx)

At this time, give students the opportunity to answer **Question #1** on the worksheet. (2-3 minutes)

Encourage the students to write their thoughts as well as identifying some new ideas they heard from their classmates.

**7.** At this time, the teacher will direct the students to the Congruence Theorems interactive software found on the Thinkfinity website and open up the same website in the front of the room for demonstration purposes.  If a classroom set of computers is not available, the teacher can use the computer in the front of the room and the students can follow the teacher through the website.

[**Congruence Theorem Software**](http://websites.pdesas.org/shirlann_finch/2010/3/17/27762/content.aspx)

The teacher will direct the students to open the Instructions and Exploration menus to locate the instructions for working through and navigating the software.  The teacher will proceed through the same steps on the computer in the front of the room.

The students will read the Instructions to themselves as the teacher reads out loud. Allow 5-10 minutes for the teacher and the students to read through the directions and allow the students to explore the software. It may be necessary to find a similar program that allows for switch access or to create a similar program using assistive technology programs such as IntelliSuites. As the students are investigating the necessary skills to manipulate the parts of a triangle, the teacher should be walking around the room and giving appropriate suggestions when a student appears frustrated.  After the students have had an opportunity to become familiar with constructing a triangle, ask the students if they have any questions.  It may be necessary for the teacher to demonstrate certain skills or help some of the students individually. This is also an excellent opportunity for students with more computer experience to help their peers.

**8.** After free-exploration, the teacher will instruct the students to scroll down until only the Exploration section and the interactive screen are on their monitors.  The teacher will proceed through the same step on the computer in the front of the room. The students and the teacher will now work together through the sample problem under the Exploration heading. After each new step, give students ample time to complete the action on their individual computers. Do not move on until all the students have given a "thumbs up!"

At the end of the exploration, there is a question:  "Given two triangles for which two sides and an adjacent angle are congruent, are the triangles congruent?  Explain."  Ask for volunteers to share their answers.  You will want to provide the student the relevant tactile vocabulary words as this question is asked as well as object or tactile representations that illustrate the two sides and adjacent angles of two triangles.

**9.** Next, as you demonstrate in the front of the room, have the students move one triangle so it lies on top of the other triangle. **Students might work in pairs to compare triangles. Students may use manipulatives to compare congruent triangles, or they may use an assistive technology program that allows the student to manipulate two triangles to place on top of the other using a switch or adaptive keyboard.** The teacher will ask the students if all the corresponding parts of the two triangles are congruent. (Yes).  The teacher will continue with the question does this fit the definition of congruent triangles? (Yes).  To complete the sample exploration, the teacher will read the question at the bottom of the monitor:  "Are these triangles congruent?" click YES and have the students do the same. If the students are using manipulatives to determine if pairs of triangles are congruent, they can sort the pairs in collection baskets labeled with the tactile representations or students may use an assistive technology program that presents two triangles and the student uses an adaptive keyboard that is divided into two cells populated with the tactile representations of Yes and No or Congruent/Not Congruent to label or sort the pairs of triangles.

The students will now click the Reset button and determine if they always get the same results.  (With this combination they will always get the same result.)

**10.** The students will now return to the worksheet and read **Question #2** as the teacher reads out loud.  Using the results from the sample exploration, the teacher will illustrate on the board the appropriate answers in each column for "1)” on the chart. Students may need to include tactile representations with sides and angles marked based on each scenario presented in column “1” of the chart.

|  |  |  |  |
| --- | --- | --- | --- |
| Name the parts you checked on the software | Is the second triangle congruent to the first? | Can you make a triangle that is not congruent with the original? | Which parts are corresponding? |
| 1)   side, side, angle or AC, AB,  <A |  |  |  |
| 2) side, side, angle or TZ, TR,  <T | YES | No | AC & TZ; AB & TR;  <A & <T |

The students are now ready to work through the interactive software on their own and record their results on the worksheet.

Allow students 15-20 minutes to work independently. **Give students the following combinations to explore and record observations on their worksheet. Provide the tactile representations of the relevant vocabulary word combinations below.**

     1)  Side-Side-Side          (SSS)

     2)  Side-Angle-Side        (SAS)

     3)  Angle-Side-Angle       (ASA)

     4)  Angle-Angle-Side       (AAS)

**11.** Next, the teacher will organize the students into small groups (a maximum of 4 students is recommended).  The students will compare their results and answer **Question #3.** (5 minutes)

At the end of class, have each group share a conclusion (successful and unsuccessful).  The teacher will record (on chalk board, white board, or large post-it-note) the successful combinations of triangle parts the students discovered were needed to make unique triangles.  Continue until all the following combinations have been identified.  It may be necessary to demonstrate examples that were not discovered.  Have students write the successful combinations for the answer to **Question #4.**

     1)  Side-Side-Side          (SSS)

     2)  Side-Angle-Side        (SAS)

     3)  Angle-Side-Angle       (ASA)

     4)  Angle-Angle-Side       (AAS)

     5)  Hypotenuse-Leg        (HL-the special case of Angle-Side-Side!)

Note:  The students may not discover Hypotenuse-Leg, on their own. The students can now answer **Question #5** and finish the worksheet. Collect the worksheets.