

University of Scranton  
ACM Student Chapter / Computing Sciences Department  
20th Annual High School Programming Contest (2010)

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**Problem 2: The  $3n + 1$  Problem**

Given a positive integer  $k$ , the  $3n + 1$  sequence starting at  $k$  has as its first element  $k$  itself. For any particular element  $m$  in the sequence, where  $m \neq 1$ , the next element is either  $3m + 1$  (if  $m$  is odd) or else  $m/2$  (if  $m$  is even). The sequence ends with the first occurrence of 1.

For example, the  $3n + 1$  sequence starting at 7 is

7 22 11 34 17 52 26 13 40 20 10 5 16 8 4 2 1

This sequence has length 17.

The *Collatz conjecture* states that for every  $k$ , the  $3n + 1$  sequence starting at  $k$  has finite length. Despite the efforts of many mathematicians over many years, this conjecture has yet to be proved or disproved.

Develop a program that, given a positive integer  $k$ , computes the length of the  $3n + 1$  sequence starting at  $k$ , as well as the largest value in that sequence.

*Hint:* If  $a$  and  $b$  are positive integers, the expression  $a \% b$  (or  $a \bmod b$  in some programming languages) produces the remainder of the division of  $a$  by  $b$ . (E.g.,  $13 \% 5$  yields 3;  $28 \% 7$  yields 0.)

**Input:** The first line contains a positive integer  $n$  indicating how many instances of the problem are described on the succeeding  $n$  lines. Each instance of the problem is described on a single line containing a single positive integer.

**Output:** For each positive integer  $k$  given as input, produce a message that identifies  $k$ , the length of the  $3n + 1$  sequence starting at  $k$ , and the maximum value in that sequence. See the sample output below for proper formatting.

Sample input:	Resultant output:
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4	For 7, sequence length is 17 and maximum value is 52.
7	For 40, sequence length is 9 and maximum value is 40.
40	For 1, sequence length is 1 and maximum value is 1.
1	For 3456, sequence length is 119 and maximum value is 9232.
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