University of Scranton
ACM Student Chapter / Computing Sciences Department
20th Annual High School Programming Contest (2010)

Problem 6: Base k Addition

Most peoples of the world use the *decimal* (or *base ten*) numeral system, in which the ten symbols 0, 1, 2, ..., 9, called the decimal *digits*, are used for forming numerals. The contribution made by each digit in a numeral depends not only upon its value (e.g., 5 vs. 7) but also upon its position within the numeral. The rightmost position is the 1's column; moving to the left from there we encounter the 10's, 100's, 1000's, etc., columns. Note that these are the powers of ten. For example, the decimal numeral 7204 corresponds to the sum

$$7204_{10} = (7 \cdot 10^3) + (2 \cdot 10^2) + (0 \cdot 10^1) + (4 \cdot 10^0)$$

(Notice that, to indicate a numeral's base explicitly, we put it as a subscript to the right of the numeral.)

There is nothing special about using 10 as the base, however. In a base 5 numeral, for example, only the digits 0 through 4 may appear, and in such a numeral we find the 1's, 5's, 25's, 125's, etc., columns, corresponding to the powers of five. Hence, the base 5 numeral 3042 represents the number given by the sum

$$(3 \cdot 5^3) + (0 \cdot 5^2) + (4 \cdot 5^1) + (2 \cdot 5^0)$$

in a manner completely analogous to the decimal numeral 7204 above.

Performing addition in base 5 (or in any other base) is completely analogous to how it is done in base 10, too. For example, in base 5 we have

```
   44032
 +  1341
  ------
  100423
```

In the 1's column, we have $2_5 + 1_5 = 2_{10} + 1_{10} = 3_{10} = 3_5$, so we record a 3 in that column. In the 5's column, we have $3_5 + 4_5 = 3_{10} + 4_{10} = 7_{10} = 12_5$, so we record 2 in that column and carry the 1 to the 25's column. In the 25's column, we have (remembering the incoming carry) $1_5 + 0_5 + 3_5 = 1_{10} + 0_{10} + 3_{10} = 4_{10} = 4_5$, so we record 4 there. In the 125's column, we have $4_5 + 1_5 = 4_{10} + 1_{10} = 5_{10} = 10_5$, so we record 0 and carry the 1. And so on.

Develop a program that, given as input an integer k, with $2 \leq k \leq 9$, and two base k numerals, calculates their sum and expresses it as a base k numeral.

Input: The first line contains a positive integer n indicating how many instances of the problem are to be solved. Each instance is described on three lines, the first of which contains the base and the next two of which contain two numerals of that base, one per line, each having at most thirty digits.

Warning: The judges' test data will include numerals representing numbers that are not in the range of values covered by intrinsic data types (e.g., `int`, `long`, `float`, `Integer`, `Real`) in languages such as C, C++, Java, or Pascal.

Output: For each pair of numerals to be added, generate a single line of output that identifies their base, the two numerals (separated by a plus sign), an equals sign, and their sum. (See the sample output below.)

Sample input:	Resultant output:
4	In base 10, 33468294890 + 7502 = 33468302392
10	In base 2, 111110001 + 110100101010011 = 110101101000100
33468294890	In base 6, 543052 + 242441 = 1225533
7502	In base 9, 543052 + 242441 = 785503
2	
111110001	
110100101010011	
6	
543052	
242441	
9	
543052	
242441	