**Arithmetic Sequences**

When given the sequence $-9, -6, -3, 0, 3, 6,…$ we can determine the explicit rule using two different approaches. For either approach, a table of values will be helpful.

|  |  |
| --- | --- |
| *x* | *y* |
| 1 | −9 |
| 2 | −6 |
| 3 | −3 |
| 4 | 0 |
| 5 | 3 |
| 6 | 6 |

**Approach #1**

The first approach involves finding the slope and *y*-intercept and writing the equation of the line. Let’s first find the slope. We will pick two points to substitute into our slope formula. Doing so gives:

$$m=\frac{y\_{2}-y\_{1}}{x\_{2}-x\_{1}}$$

$$=\frac{-6-(-9)}{2-1}$$

$$=\frac{-6+9}{1}$$

$$=\frac{3}{1}$$

$$=3$$

With the slope determined, we now need to find the y-intercept. The traditional way to find the y-intercept is to substitute an x-value and y-value of a coordinate pair into the slope-intercept form of a line, using the slope we just found. Using the values from the first coordinate pair, we have:

$$y=mx+b$$

$$-9=3\left(1\right)+b$$

$$-9=3+b$$

$$-9-3=b$$

$$b=-12$$

**Note:** We could’ve also found the y-intercept from the table. The *y*-intercept is the point whereby the *x*-value is 0. If you find one less than an x-value of 1, you have an *x*-value of 0. The rate of change is +3. Therefore, if you go in the opposite direction, you will subtract 3, and $-9-3=-12$. The rule for this sequence is $y=3x-12$.

**Approach #2**

We can also use the formula for finding the *n*th term of an arithmetic sequence. This sequence is arithmetic because there is a constant rate of change, or a constant difference between terms.

The formula for finding the *n*th term of an arithmetic sequence is:

$$a\_{n}=a+\left(n-1\right)d$$

 where *a* represents the first term and *d* is the difference.

Substituting our values for *a* and *d*, we obtain the following:

$$a\_{n}=-9+\left(n-1\right)3$$

$$=-9+3n-3$$

$$=3n-12$$

Again, we find the same rule, with the exception of the letter variable used.

For example, what is the 17th term in the sequence –23, –20, –17, –14,…?

For this sequence, the difference is 3 and the first term is –23. Substituting,

$$a\_{n}=a+\left(n-1\right)d$$

$a\_{n}= $–23 + (16) 3

$a\_{n}= $25